Effects of the Spin-Orbit and Tensor Interactions on the $M1$ and $E2$ Excitations in Light Nuclei

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The effects of varying the spin-orbit and tensor components of a realistic interaction on $M1$ excitation rates and $B(E2)'s$ are studied on nuclei in the $0p$ and $1s - 0d$ shells. Not only the total $M1$ but also the spin and orbital parts separately are studied. The single-particle energies are first calculated with the same interaction that is used between the valence nucleons. Later this stringent condition is relaxed somewhat and the $1s$ level is raised relative to $0d$. For nuclei up to ^{28}Si, much better results i.e stronger $B(M1)$ rates are obtained by increasing the strength of the spin-orbit interaction relative to the free value. This is probably also true for ^{32}S, but ^{36}Ar presents some difficulties. The effects of weakening the tensor interaction are also studied. On a more subtle level, the optimum spin-orbit interaction in the lower half of the $s - d$ shell, as far as $M1$ excitations are concerned, is substantially larger than the difference $E(J = 3/2^+)_1 - E(J = 5/2^+)_1 = 5.2 \ MeV$ in ^{17}O. A larger spin-orbit splitting is also needed to destroy the triaxiality in ^{22}Ne. Also studied are how much $M1$ orbital and spin strength lies in an observable region and how much is buried in the grass at higher energies. It is noted that for many nuclei the sum $B(M1)_{orbital} + B(M1)_{spin}$ is very close to $B(M1)_{total}$, indicating that the summed cross terms are very small.

I. INTRODUCTION

Our purpose is to see how various components of the realistic nucleon-nucleon interaction affect nuclear properties. In particular, we shall investigate magnetic dipole excitations to $J = 1^+$ states of even-even nuclei -these are spin and orbital modes, and as a counter point we shall look at low lying 2^+ states which for deformed open shell nuclei have very little spin content. We will examine selected nuclei in the $0p$ and $1s - 0d$ shell.

At first thought it may seem that such an investigation is unnecessary because excellent fits have been obtained by Cohen and Kurath [1] in the p shell and by Wildenthal and Brown [2] in the $1s - 0d$ shell. These works are of extreme importance if for no other reason than that they showed that the shell model works well beyond many peoples expectations -well enough in fact so that the authors in collaboration with the experimental groups could meaningfully go one step further and discuss effects of exchange currents on $M1$'s in a systematic way [3,4]. The works brought order out of the myriad of energy levels that were discussed by experimentalists.

However, the truly excellent fits were obtained with empirical effective interactions e.g. the Wildenthal interaction in the sd shell [5]. Such an interaction implicitly has in it several effects e.g. core polarization, relativistic phenomenology etc. It is impossible to sort out these effects from an empirical interaction expressed as hundreds of matrix elements.

Our intention is to start with a realistic interaction which will admittedly not give as good a fit to the data as the above mentioned empirical interactions. But the discrepancies will show what is missing and what has to be done. The interaction that we use -the (x, y) interaction- has been discussed elsewhere [6]. Suffice it to say that it is written as follows:

$$V(r) = V_c(r) + xV_{s.o.} + yV_t$$

where *s.o.* stands for the two-body spin-orbit interaction and t for the tensor interaction and $V_c(r)$ is everything else especially the central interaction. For $x = 1$, $y = 1$, the interaction gives a good fit to the Bonn A matrix elements [6]. We can study the effects of changing the spin-orbit and tensor interactions by varying x and y.

Most calculations which claim to use realistic interactions are really doing hybrid calculations in which the residual two-body matrix elements are carefully calculated but the single-particle energies are obtained from experiment. Here on the other hand we shall use the same interaction to *calculate* the single-particle matrix elements as was used to calculate the residual interaction. We feel this is the only way to *really* find out the effects of such an interaction. However, to overcome the deficiencies of using harmonic oscillator wavefunctions, we will also consider one case in which the relative spacing of the $1s$ and $0d$ single-particle energies are changed (likewise for p and $0f$).

We shall perform calculations with four sets of (x, y): (1,1), (1.5,1), (1.,0.5) and (1.5,0.5). We are motivated in these choices by the ideas presented by many that the spin-orbit interaction inside a nucleus should be stronger than what is deduced form nucleon-nucleon scattering and the tensor interaction weaker. We shall also consider one more case in which, for the $1s - 0d$ shell, the $1s$ single-particle energy is moved up by $2\ MeV$ relative to the $0d$ centroid ($\Delta\epsilon_s = 2\ MeV$). This is to overcome the defects of using harmonic oscillator wavefunctions, and is equivalent to the Nilsson prescription of adding a $D\vec{l}\cdot\vec{l}$ term to the Hamiltonian.

A stronger spin-orbit interaction arises for example in the Walecka model, also known as Dirac phenomenology [7]. There is a parameter in the theory called the Dirac effective mass which arises from sigma exchange. The ratio m_D/m is less than one and the spin-orbit interaction is inversely proportional to m_D/m. In another formulation of Wiring et. al., the stronger spin-orbit interaction comes from a three-body interaction [8].

Concerning the weakening of the tensor interaction inside a nucleus, two basic ideas have been advanced. One is the universal scaling argument by G.E. Brown and collaborators [9]. The basic idea is that all mesons inside the nucleus except for the pion become less massive. Hence the range of the nucleon-nucleon interaction due to a given boson exchange becomes larger. In particular, the ρ meson exchange will be of larger range. This gives a larger repulsive contribution to the tensor interaction which will cancel the attraction due to the pion. The increased cancellation will result in a weaker tensor interaction.

On the other hand Fayache, Zheng and Zamick have advocated a self-weakening mechanism for the tensor interaction [10]. They show that the effects of higher-shell admixtures makes the tensor interaction in a $\Delta N = 0$ (one major shell) space appear weaker. Thus, at least for *nuclear structure calculations*, no mechanism beyond higher shell admixtures seems to be needed.

However, the theory of Brown et. al. [9] also pertains to nucleon-nucleus scattering problems at intermediate to high energies. Here the anomalies for the q^2 behaviour of several Wolfenstein parameters can be explained by the universal scaling arguments, but it has not been demonstrated that better nuclear structure calculations would also resolve these anomalies.

II. THE MAGNETIC DIPOLE OPERATOR

In what follows we discuss present calculations for what we call $B(M1)_{physical}$, $B(M1)_{orbital}$ and $B(M1)_{spin}$. In terms of a magnetic dipole operator, the $B(M1)$ is defined as

$$\frac{1}{(2J_i + 1)} \sum_{M_f,\mu,M_i} |\langle \Psi^{J_f}_{M_f}\ A\ _\mu\ \Psi^{J_i}_{M_i}\rangle|^2$$

where

$$\vec{A} = \sqrt{3/4\pi}\left[\sum_{protons} (g_{l\pi}\vec{l}_i + g_{s\pi}\vec{s}_i) + \sum_{neutrons} (g_{l\nu}\vec{l}_i + g_{s\nu}\vec{s}_i)\right]$$

For the three cases considered, the following parameters were used:

Physical $g_{l\pi} = 1$ $g_{s\pi} = 5.586$ $g_{l\nu} = 0$ $g_{s\nu} = -3.826$

Orbital $g_{l\pi} = 1$ $g_{s\pi} = 0$ $g_{l\nu} = 0$ $g_{s\nu} = 0$

Spin $g_{l\pi} = 0$ $g_{s\pi} = 5.586$ $g_{l\nu} = 0$ $g_{s\nu} = -3.826$

In other words, for the $B(M1)_{orbital}$ we set the spin g factors to zero and for $B(M1)_{spin}$ we set the orbital g factors to zero.

One can also write the magnetic moment operator as $\vec{A} = \vec{A}_{scalar} + \vec{A}_{vector}\tau_z$ where $\tau_z = +1$ for the proton and -1 for the neutron. For the physical operator,

$$g_s(scalar) = \frac{g_{s\pi} + g_{s\nu}}{2} = 0.88$$

and

$$g_s(vector) = \frac{g_{s\pi} - g_{s\nu}}{2} = 4.706$$

Both $g_l(scalar)$ and $g_l(vector)$ are equal to 0.5. Because $g_s(vector)$ is much larger than $g_s(scalar)$, $T = 0$ to $T = 1$ transitions in $N = Z$ nuclei tend to be much larger that $T = 0 \rightarrow T = 0$ transitions. The ratio $(\frac{g_s(vector)}{g_s(scalar)})^2$ is equal to 81.

There are some interesting limiting cases for $M1$ matrix elements. For example, in the $SU(4)$ limit, all spin matrix transition matrix elements from the $J = 0^+$ ground state (both isoscalar and isovector) will vanish. The isoscalar orbital transition will also vanish. All that is left is the isovector orbital excitations known as scissors modes.

For strongly deformed nuclei one approaches this limit. The nuclei in the lower part of the $s - d$ shell e.g. ^{20}Ne and ^{24}Mg are strongly deformed and so we should look for such selection rules there. However, the spin contributions will not be negligible because $\frac{g_s(vector)}{g_l(vector)} = 9.41$, and even if the spin matrix elements are strongly suppressed they start out with a big advantage.

III. THE SINGLE-PARTICLE ENERGIES

As mentioned in the introduction, we do not perform a hybrid calculation. We first obtain the single-particle energies with the same interaction that is used for the two-particle matrix elements. More to the point, in our shell model, we perform a calculation for say ^{20}Ne not merely as four nucleons in the $1s - 0d$ shell but rather as twenty nucleons with sixteen in the closed $0s$ and $0p$ shells. This allows for valence-core interactions which implicitly generate the single-particle energies. (In a later section we relax this stringent condition by moving the $1s$ single-particle level relative to $0d$ in order to overcome the defects of using harmonic oscillator wavefunctions).

In Table I we present the single-particle energies obtained for the (x, y) interaction with $x = 1$, $y = 1$ in both a small space and a large space. By the former we mean that the ^{16}O core is inert. In the large space, we allow $2\hbar\omega$ excitations in the shell model diagonalization and we set the lowest state to zero [11].

We see one defect in that the lower l states come down too low relative to the higher l states. Whereas in the calculation the $1s_{1/2}$ comes below $0d_{5/2}$, for ^{17}O we know it's the other way around. Likewise the $1p_{3/2}$ comes 0.94 MeV below $0f_{7/2}$ whereas experimentally the $1p_{3/2}$ is about 2 MeV above $0f_{7/2}$. This tendency of the low l to come too low has the effect of making nuclei more deformed than they are experimentally.

The calculated spin-orbit splitting for ^{15}O in the small space is 5.07 MeV, somewhat smaller than the splitting between the $J = 3/2^-$ excited state of ^{15}O and the $J = 1/2^-$ ground state. The latter is 6.1 MeV. For ^{17}O the

splitting $J = 3/2^- - J = 1/2^-$ is 5.562 MeV in the calculation, which compares well with the empirical splitting of 5.2 MeV.

Going to the large space does not increase the splitting significantly. For $A = 15$ the value is 5.679 MeV while for $A = 17$ it is 5.662 MeV.

An interesting question to ask is whether the empirical procedure above of identifying the spin-orbit splitting in say ^{17}O with $E(J = 3/2^-) - E(J = 5/2^-)$ is valid. We shall discuss this in the context of the systematics of the magnetic dipole transitions that we calculate.

IV. RESULTS FOR $M1$ AND $E2$ EXCITATION RATES AS A FUNCTION OF X AND Y

(a) The Effects of the Spin-Orbit and Tensor Interactions on the Spin $B(M1)$

For all nuclei considered, the effect of increasing the spin-orbit strength parameter x from 1 to 1.5 causes the spin $B(M1)$ (also denoted by B_σ -last column of Table II) to increase. In ^{12}C the increase (for $y = 1$) is from 0.91 to 2.07; in ^{20}Ne from 0.59 to 1.28; in ^{24}Mg from 1.73 to 3.35, in ^{32}S from 2.72 to 8.52; and in ^{36}Ar from 2.25 to 5.20. The change is especially large in ^{32}S.

The increase in B_σ can be understood by noting that this quantity increases as we go from the LS limit to the $j - j$ limit. Indeed, in the $SU(4)$ limit, the spin $B(M1)$ will vanish.

In all cases considered, decreasing the tensor interaction also causes the spin $B(M1)$ to increase. This is consistent with the old observation of C.W. Wong [12] that the tensor interaction in an open shell nucleus acts to some extent like a spin-orbit interaction of the opposite sign as the main spin-orbit term. Thus, in ^{24}Mg, when we go from $x = 1$ $y = 1$ to $x = 1$ $y = 0.5$ the $B(M1)_{spin}$ (in units of μ_N^2) goes from 1.73 to 1.94; and in going from $x = 1.5$ $y = 1$ to $x = 1.5$ $y = 0.5$, we find that $B(M1)_{spin}$ increases from 3.35 to 4.04. There is a similar behaviour for all other nuclei considered. The effect of varying the tensor strength y is not quite as dramatic as that of varying the spin-orbit strength x.

It should be emphasized that there are basic differences between the tensor and the spin-orbit interactions. For a closed LS plus or minus one nucleon, the tensor interaction gives no contribution to the single-particle energy in first order whilst the two-body spin orbit interaction does. The latter gives the majority amount of the spin-orbit splitting e.g. of $d_{3/2}$ and $d_{5/2}$ in ^{17}O. What we are discussing in the above paragraph is an open shell effect.

To summarize, for the cases considered we get the smallest value of $B(M1)_{spin}$ (and also $B(M1)_{physical}$) for $x = 1$ $y = 1$ and the largest value for $x = 1.5$ $y = 0.5$ i.e. a stronger spin-orbit and a weaker tensor interaction than what is furnished by large G matrix elements.

(b) The Effects of the Spin-Orbit and Tensor Interactions on the Orbital $B(M1)$

The effect of varying x and y on the orbital summed strength is more complicated than for the spin $B(M1)$. The effects are however not insignificant.

For $N = Z$ nuclei, we find that the effects become more important as we go the end of the shell. For ^{20}Ne, the effect of changing x and y are very small. For the four sets of x and y, the values of B_l are 0.95, 1.09, 0.96, and 1.10 (in the order as shown in Table II). For ^{32}S, the results are 2.32, 1.57, 2.17 and 1.15; and for ^{36}Ar they are 1.44, 0.95, 1.33 and 0.87. We also see that increasing the spin-orbit strength for ^{24}Mg, ^{32}S and ^{36}Ar decreases the summed orbital strength. Decreasing y (the tensor strength) in some cases (e.g. ^{20}Ne) slightly increases the orbital strength, but in most cases, it causes a decrease in this strength. For example in ^{32}S, going from $x = 1$ $y = 1$ to $x = 1$ $y = 0.5$ the orbital strength decreases slightly from 2.32 to 2.17; whilst from $x = 1.5$ $y = 1$ to $x = 1.5$ $y = 0.5$ the orbital strength decreases from 1.57 to 1.15 μ_N^2.

(c) The Physical $B(M1)$ -additional observations

The dependence of the physical $B(M1)$ on x and y is qualitatively similar to that of $B(M1)_{spin}$. In general, increasing x and decreasing y will cause the summed strength $B(M1)_{physical}$ to increase.

Recalling that $\frac{g_s(isovector)}{g_l(isovector)} = 9.41$, this fact alone would cause $B(M1)_{spin}$ to be about 85 times larger than $B(M1)_{orbital}$. However, overall in Table II, $B(M1)_{orbital}$ and $B(M1)_{spin}$ are comparable in magnitude. For example, for $x = 1$ $y = 1$ in ^{24}Mg, $B(M1)_{orbital} = 2.08$ and $B(M1)_{spin} = 1.73$. It is true that when we change to $x = 1.5$ $y = 0.5$ the $B(M1)_{spin}$ becomes larger than $B(M1)_{orbital}$ (4.04 vs. 1.69), but the ratio is much less than 85.

These and other numbers in Table II confirm what was said in a previous section -that the spin matrix elements are strongly suppressed because in deformed nuclei one moves away from the jj limit towards the LS $SU(4)$ limit.

(d) The Relationship of the Physical $B(M1)$ to the Sum of the Orbital $B(M1)$ and the Spin $B(M1)$

Let us consider the $B(M1)$ transition from the ground state 0^+ to any single 1^+ state. Then the physical $B(M1)$ will be the sum of the orbital $B(M1)$ and the spin $B(M1)$ plus their cross terms which in principle may be positive or negative. In Table III the spin, orbital and physical $B(M1)$ results each represent the sum (over all the 1^+ states in our space) of the corresponding single-state $B(M1)$'s. Let us now treat the ^{22}Ne results as one unit, summing the $T = 1 \rightarrow T = 1$ (both isovector and isoscalar) and $T = 1 \rightarrow T = 2$ results. Then for all the nuclei that we consider and for all the x, y combinations (all 28 cases except for the $x = 1$ $y = 1$ case in ^{44}Ti with a difference of less than 1%), the physical $B(M1)$ is *less* than the sum of the spin $B(M1)$ and orbital $B(M1)$. Remarkably, in all the ^{20}Ne, ^{22}Ne and ^{44}Ti cases the difference is less than 3%; for ^{24}Mg less than 6%; all these cases are in the first half of the major shell. For the other three cases ^{12}C, ^{32}S and ^{36}Ar (all in the second half of major shells), the difference is still always less than 20%, with the physical $B(M1)$ being less than the sum of the other two. Just to illustrate, in ^{20}Ne the four respective spin $B(M1)$ plus orbital $B(M1)$ sums (i.e. for the four x, y combinations) are 1.54, 2.37, 1.59 and 2.59; the corresponding physical $B(M1)$ values are 1.53, 2.34, 1.57 and 2.54!

(e) Summed Strengths for Magnetic Dipole Excitations

In Tables III and IV we compare the summed strength (somewhat arbitrarily) to the first ten states, and we also give the total sum. We limit ourselves to the case $x = 1.5$, $y = 0.5$. In Table IV we also have $x = 1.5$ $y = 0.5$ but we have $\Delta\epsilon_s = 2$ MeV.

Concerning the orbital strength we find in some cases a significant difference between the summed strength to all states and the sum to the first ten states. As seen in Table III for ^{20}Ne the difference is not so large (1.10 vs. 0.90), for ^{22}Ne $T = 1 \rightarrow T = 1$ there is a factor of two difference (0.46 vs. 0.23). For ^{24}Mg and ^{32}S the differences are also large (2.02 vs. 1.39 and 1.15 vs. 0.53 respectively).

This is of relevance to works on relations between summed orbital (scissors mode) strength and $B(E2)_{0_1 \rightarrow 2_1^+}$ in deformed nuclei. What is precisely meant by the summed $M1$ orbital strength? Experimentally it is very difficult if not impossible to pick up the orbital "grass" which presumably sets in at the very least by the time we have come to the first ten states if not earlier. What then do the sum rules mean if experimentally not all orbital strength is included?

(f) Comprehensive List of $M1$ Rates

In Tables V and VI we give a comprehensive list of calculated values of excitation energy (MeV), B_l, B_σ and $B(M1)$ in units of μ_N^2 for the four sets of (x, y) -(1,1), (1.5,1), (1,0.5) and (1.5,0.5). In order not to drown ourselves in details, we limit Table V to states for which $B(M1)$ is greater or equal to $0.3\mu_N^2$ (there is one exception in ^{20}Ne $x = 1$, $y = 0.5$ where the strengths to two nearly degenerate states is given). However, there are other states of interest which are put into Table VI. For example, there are states with substantial orbital strengths and spin strengths but the destructive interference of the orbital and spin amplitudes causes $B(M1)$ to be very small. In the case of ^{36}Ar, such an 'invisible' state is lower in energy than the lowest strongly excited state.

On the whole, the behaviour in Table V is similar to that of Table II where the total summed strength is given. That is to say, in going from (x, y)=(1,1) to (1.5,1) the values of B_σ and $B(M1)$ increase. In making the analysis

however, we have to take note that the fragmentation can be different for different values of (x, y).

The fragmentation mentioned above is greater in some nuclei than in others. Whereas for ^8Be and ^{12}C there is only one state with a calculated value of $B(M1)$ greater than 0.3 μ_N^2, in ^{32}S and in ^{36}Ar there are four (with $x = 1.5$, $y = 0.5$). In ^{32}S states calculated to be at 10.05, 11.21, 13.01 and 13.81 MeV have values of $B(M1)$ 2.91, 5.34, 0.68 and 0.44 μ_N^2 respectively. In ^{36}Ar states at 11.91, 12.97, 13.73 and 17.65 MeV have $B(M1)$ values of 2.62, 1.46, 0.55 and 0.49 μ_N^2. For ^{36}Ar the orbital strengths to these states is very small -it is mainly spin strength.

Table VI shows states with very weak $B(M1)$'s but for which B_l and/or B_σ are fairly large. The destructive interference between the orbital and spin *amplitudes* causes $B(M1)$ to be very small. In the following discussion, we will be referring to calculated results for $x = 1.5$, $y = 0.5$ (not experiment).

In some cases it is the lowest 1^+ $T = 1$ state for which this destructive interference occurs. This is especially true for ^{36}Ar where the lowest calculated $J = 1^+$ $T = 1$ state for $x = 1.5$ $y = 0.5$ is at 7.80 MeV with $B(M1)$ negligibly small ($< 5 \times 10^{-3} \mu_N^2$). The next 1^+ $T = 1$ state is over 4 MeV higher in energy. This state is at 11.91 MeV and has $B(M1) = 2.62 \mu_N^2$. The 7.80 MeV state has significant B_l and B_σ values: $0.305 \mu_N^2$ and $0.320 \mu_N^2$ respectively. The amplitudes are nearly equal but of opposite sign, thus causing $B(M1)$ to nearly vanish.

In ^8Be and ^{20}Ne, the lowest states have the largest $B(M1)$ values, but not too high above these are the 'invisible' states with substantial B_l and B_σ but negligible $B(M1)$. Thus from a pure (e, e') experiment one will miss considerable orbital strength which is buried in these invisible states. Only with a combination of (e, e') and another probe e.g. (p, p') can one unravel the spin and orbital content of the $M1$ strength.

(g) $B(E2)$ **Rates**

In Table VII we present results for summed isoscalar and isovector $B(E2)$ for the four different sets of (x, y). The general $E2$ operator is

$$\sum_\pi e_p r^2 Y_{2,\mu} \;+\; \sum_\nu e_n r^2 Y_{2,\mu}$$

For the isoscalar case we take $e_p = 1.5$, $e_n = 0.5$ and for the isovector case $e_p = 1$, $e_n = -1$.

It should be noted that, for transitions $J = 0^+$ $T = 0$ $\rightarrow J = 2^+$ $T = 1$, the overwhelming amount of the strength goes to the lowest $J = 2^+$ state. For example, for ^{20}Ne (using $x = 1$, $y = 1$) the value of $B(E2) \uparrow$ to the lowest 2^+ state at 2.92 MeV is 299.4 $e^2 fm^4$ whilst the total sum is 315.1 $e^2 fm^4$. This seems to be true for all nuclei except ^{22}Ne where there is considerable fragmentation between the first two 2^+ states. For $x = 1$, $y = 1$, the value of $B(E2)$ is 198.9 $e^2 fm^4$ to the 2_1^+ state at 2.27 MeV and 113.4 $e^2 fm^4$ to the 3.10 MeV state. The sum of these two strengths (312.3 $e^2 fm^4$) is however close to the total strength (352.7 $e^2 fm^4$).

It should be noted that for most nuclei considered here, the sensitivity of $B(E2)$ to x and y is much less than it is for isovector $B(M1)'s$. For example, for ^{20}Ne the four values of $B(E2)_{isoscalar}$ are 315.1, 313.5, 316.0 and 311.1 $e^2 fm^4$ respectively.

This can be understood from the fact that the strong central interaction causes the low lying 2^+ states to be dominantly $S = 0$ states. The expectation values of the spin-orbit and tensor interactions for pure $S = 0$ states are zero.

For the ^{32}S nucleus however, there is considerable sensitivity of the $B(E2)_{isoscalar}$ to x and y. For this nucleus there must be a lot of the $S = 1$ component in the ground state.

V. EFFECTS OF RAISING THE $1S$ LEVEL WITH RESPECT TO THE $0D$ LEVEL

In our calculations for nuclei in the $1s-0d$ shell the calculated physical $B(M1)$ values, especially with the $x = 1$ $y = 1$ interaction, were often smaller than the corresponding measured experimental values. This suggested to us that for these nuclei we should further investigate one additional aspect of our calculations, namely the energy separation of

the single-particle $1s$ and $0d$ levels. From Table I it appears that the $1s$ level is too low in energy relative to the centroid of the $d_{5/2}$ and $d_{3/2}$ levels. The fault may lie with the fact that we are using harmonic oscillator wavefunctions to evaluate matrix elements of the two-body interaction.

To study the above effect we redid our calculations with the $x = 1.5\ y = 0.5$ interaction for the $1s - 0d$ nuclei with the $1s$ level raised by an *additional* $2\ MeV$ above the $0d$ level. This correction is equivalent to adding a term $D\vec{l} \cdot \vec{l}$ to the single-particle spectrum, as was done in the Nilsson model. To calculate the best value of the energy difference to adopt, we would have proceeded as follows. Our calculations for the single-particle energies in ^{17}O using the $x = 1\ y = 1$ interaction (see Table I) yielded the following energies (in MeV): $\epsilon_{s_{1/2}} = 0$, $\epsilon_{0d_{5/2}} = 0.119$, $\epsilon_{0d_{3/2}} = 5.681$. These place the centroid of the $0d$ levels at $2.34\ MeV$ above the $1s$. Experimentally, we find the single-particle energies (in MeV) in ^{17}O to be $\epsilon_{d_{5/2}} = 0$, $\epsilon_{1s_{1/2}} = 0.871$, and $\epsilon_{d_{3/2}} = 5.09$. This places the $0d$ centroid at $2.03\ MeV$, or $1.16\ MeV$ above the $1s$. Accordingly, the $1s$ should have been raised by $(2.35\text{-}1.16)=1.19\ MeV$. We chose however chose $2\ MeV$ rather than $1.19\ MeV$.

In Table IV we present the effects of raising the $1s$ level relative to the $0d$ level for the $x = 1.5\ y = 0.5$ case. In this table we consider the summed $B(M1)$ values (physical, orbital and spin) for transitions from the ground state to the lowest $500\ J = 1^+\ T = 1$ states (or a smaller number if fewer states exist in our space). From the table, we see that the net effect of raising the $1s_{1/2}$ level is larger in the first half of the shell. Perhaps this is because in the second half of the shell, the $1s_{1/2}$ level plays a more minor role in excitations, being nearly fully occupied in both the ground state and the excited $J = 1^+\ T = 0$ states. In the first half of the shell, the total summed spin $M1$ strength (B_σ) increases by about 30% in three cases: ^{20}Ne, ^{22}Ne ($T = 1 \to T = 1$) and ^{24}Mg, and by 50% in ^{22}Ne ($T = 1 \to T = 2$). The summed orbital isovector $M1$ strength (B_l) increases by 20% and 30% for ^{20}Ne and ^{22}Ne respectively, but decreases by 4% for ^{24}Mg. Finally the summed physical $B(M1)$'s increase by 20% to 40% for these nuclei (^{20}Ne, ^{22}Ne and ^{24}Mg). This overall increasing trend in the strength of the $B(M1)$'s is welcome because our calculations (especially with $x = 1\ y = 1$) typically underestimate the experimental data there.

However, if we look at the summed strength to the first 10 states, there is a much bigger change in going from $x = 1.5\ y = 0.5$ to $x = 1.5\ y = 0.5\ \Delta\epsilon_s = 2\ MeV$. This can be seen from tables III and IV. In particular, for ^{24}Mg, the value of B_σ to the first 10 states (and this more or less covers the range of experimental activity) changes from $1.79\ \mu_N^2$ to $3.38\ \mu_N^2$. This brings us close to the experimental value of $5.86\ \mu_N^2$. The increase in B_σ when we increase the $1s$ single-particle energy can be easily understood. For a pure $(1s)^n$ ground state configuration there can be no $M1$ excitations. Thus, for say ^{20}Ne, when we increase $\Delta\epsilon_s$, the occupation of $1s$ goes down and the occupation of $0d$ goes up. Thus there will be an increase in B_σ.

One can also see this from the energy-weighted sum rule of Kurath [13]. He assumed that the most important part of the Hamiltonian for $M1$ spin transitions is the one-body spin-orbit interaction $-a\vec{l} \cdot \vec{s}$. Following a discussion of Zamick, Abbas and Halemane [14], the isovector $M1$ operator can be written as $\sqrt{3/16\pi}[-j_i\tau_i + (g_n - g_p + 1)s_i\tau_i]$ where $\vec{j} = \vec{l} + \vec{s}$, $g_n = -3.836\ \mu_N$ and $g_l = 5.586\ \mu_N$. Neglecting the $j_i\tau_i$ term, the energy weighted sum rule of Kurath [13] obtained by the double-commutator method is:

$$\sum (E_n - E_0)B(M1) \uparrow = \frac{3}{32\pi}a(g_n - g_p + 1)^2 \sum_l [ln_{l+1/2} - (l+1)n_{l-1/2}]$$

where n_j is the occupation of the j orbit. Clearly if $l = 0$ there will be no contribution. For the $s - d$ shell, the relevant factor is $(2n_{d5/2} - 3n_{d3/2})$.

For the $N = Z$ even-even nuclei in the second half of the shell, the effect of raising the $1s$ level by $2\ MeV$ is less dramatic, perhaps due to the possible reasons noted above. In ^{32}S and ^{36}Ar the physical $B(M1)$ sums *decrease* by 5% and 7% respectively, the orbital $B(M1)$'s increase by 4% and 8% and the spin $B(M1)$'s decrease by 5% in ^{36}Ar and by 9% in ^{32}S.

In Table V, and for the $x = 1.5\ y = 0.5$ interaction, we can see the effects of raising the $1s$ by $2\ MeV$ above the $0d$

level on the summed $B(E2)$'s (from the 0^+ ground state to up to 500 2^+ states). For the isoscalar transitions from the $J = 2^+$ $T = 0$ (which dominate by an order of magnitude), the effect is a decrease in the first half of the shell (by about 15% in ^{20}Ne, 13% in the $T = 1 \rightarrow T = 1$ transitions in $22Ne$, and by 2% in ^{24}Mg) and an increase in the second half of the shell (by 26% for ^{32}S and by 10% for ^{36}Ar). For the weaker isovector transitions, the effect is smaller; it is usually a decrease (by 11% in $20Ne$, 4% for $T = 1 \rightarrow T = 2$ in ^{22}Ne, 4% for ^{32}S and 7% in ^{36}Ar) except for a 3% increase in ^{24}Mg.

In all the five nuclei in Table V, the isoscalar $B(E2)$ sum over all the states is dominated by the $B(E2)$ from the ground state to the lowest $J = 2^+$ $T = 0$ state; that state contributes between about 75% and 90% of the total. By contrast, there is less dominance of a single state in the isovector $B(E2)$ sum (from the ground state to the $J = 2^+$ $T = 1$ states). There the state with the strongest transition contributes about 20% (^{24}Mg) to 60% (^{32}S).

VI. DISCUSSION OF INDIVIDUAL LEVELS -COMPARISON WITH EXPERIMENT

We now proceed with a more systematic discussion of individual levels and a comparison with experiment. This will involve information contained in Tables VIII up to XXVI.

A. ^8Be

In ^8Be, there is a strong $M1$ transition to the ground state from a $J = 1^+$ $T = 1$ state at 17.6 MeV from which we can infer that $B(M1) \uparrow = 0.8$ μ_N^2. It is tempting to associate this 17.6 MeV state with a scissors mode excitation. It is of interest to note that this is the *only* excited state of ^8Be that communicates with the ground state i.e. has an observed branch to it. The 2^+-to-ground $E2$ is *not* observed because the 2^+ state quickly decays into two alpha particles. We expect ^8Be to be a strongly deformed nucleus.

We find that for $(x = 1, y = 1)$ the strongest $B(M1) \uparrow$ to a 1^+ $T = 1$ state is to the lowest calculated state at 13.73 MeV with $B(M1) \uparrow = 0.71$ μ_N^2. The lowest 1^+ $T = 0$ state is at a higher energy (14.385 MeV). This inverse isospin ordering is in agreement with experiment. However the states come too low in energy compared with the experimental values (17.6 MeV and 18.5 MeV).

A spin-orbit analysis shows that the (calculated) state is not purely orbital:$B_l = 0.26$ μ_N^2 and $B_\sigma = 0.15$ μ_N^2. There is another state calculated to be nearly 3 MeV above the 13.73 MeV state with $B_l = 0.23$ μ_N^2, almost as much as for the lowest state. The value of B_σ to this state is 0.053 μ_N^2. However, destructive interference between spin and orbit causes $B(M1)$ to be negligible ($B(M1) = 0.065$ μ_N^2).

When we change from $x = 1$ $y = 1$ to $x = 1.5$ $y = 1$, B_l changes very little, but B_σ increases to 0.217 μ_N^2 and consequently $B(M1)$ increases to 0.97 μ_N^2. If we finally change to $x = 1.5$, $y = 0.5$ i.e. increase the spin-orbit and decrease the tensor, the value of $B(M1)$ increases slightly to 1.01 μ_N^2.

It should be noted that increasing the spin-orbit interaction does not increase the energy of the $J = 1^+$ state. From $x = 1$ $y = 1$ to $x = 1.5$ $y = 1$ the energy actually decreases from 13.73 MeV to 13.06 MeV. This is contrary to the behaviour of a naive spherical model where the 1^+ state is pictured as a particle-hole state of the form $d_{3/2}d_{5/2}^{-1}$. The energy of such a state would clearly increase linearly with the spin-orbit strength x. However, since ^8Be (and most of the other nuclei we will be considering here) are strongly deformed, such 'spherical' arguments do not hold water. This problem has been discussed by Zheng and Zamick [6]. Previous work on ^8Be and ^{10}Be by Fayache et. al. should be noted.

As seen from Table VI, there is also in ^8Be an 'invisible' state. For $x = 1.5$, $y = 0.5$ this state is 3.7 MeV above the lowest strong state. The calculated values of B_l, B_σ and $B(M1)$ are 0.14, 0.023, and 0.050 μ_N^2 respectively. There is substantial orbital strength. Experimentally, such a state has not been seen.

Studies of ^8Be and ^{10}Be were previously carried out by Fayache, Zamick and Sharma [15,16]. The study was limited to the $x = 1$ $y = 1$ case, but the quadrupole-quadrupole interaction was also studied. With the $Q \cdot Q$ interaction, it can be shown that the the orbital strength $B_l \uparrow$ in ^8Be is $\frac{2}{\pi}\mu_N^2 = 0.64\mu_N^2$. This is very close to the results obtained in Table II (0.64 or 0.67 μ_N^2).

B. ^{10}Be

In ^{10}Be, there is no information on $M1$ excitations. There is a strong $E2$ from the ground state to the 2_1^+ state $B(E2) \uparrow= 52$ e^2fm^4. Raman $et.al.$ deduce from this a deformation parameter $\beta = 1.13$ [17]. The experimental result is somewhat smaller than the isoscalar result of Table VII (71 e^2fm^4). It should be remembered that the theoretical results were obtained with effective charges $e_p = 1.5$ $e_n = 0.5$. There is some evidence that in the $0p$ shell, the effective charges should be somewhat smaller.

As shown by Fayache $et.al.$ [16], with a $Q \cdot Q$ interaction one could obtain analytic results for the isovector orbital strength B_l in ^{10}Be. The results were:

$$B_l(0^+ \ T = 1 \rightarrow 1^+ \ T = 1) = \frac{9}{32\pi}\mu_N^2 = 0.0895\mu_N^2$$

and

$$B_l(0^+ \ T = 1 \rightarrow 1^+ \ T = 2) = \frac{15}{32\pi}\mu_N^2 = 0.1492\mu_N^2$$

The total orbital strength $(T = 1 + T = 2)$ of $\frac{24}{32\pi}\mu_N^2$ is only 3/8 of the total orbital strength in ^8Be.

As seen in Table II, the results for isovector B_l for all four cases of the (x,y) interaction interaction seem to be in close accord with the analytic results of the $Q \cdot Q$ interaction above.

From Table V we note that the two lowest lying states fro $x = 1$ $y = 1$ are calculated to be at 6.14 MeV and 7.68 MeV with values of $B(M1)$ of 1.1 10^{-2} μ_N^2 and 1.85 μ_N^2 respectively. These states have negligible orbital content -they are spin modes not orbital modes. The calculated $T = 1$ and $T = 2$ isovector orbital strengths (i.e. scissors modes) [16] are at much higher energies ≈ 19 MeV. With a $Q \cdot Q$ interaction, the $T = 1$ and $T = 2$ orbital strengths are degenerate. For $x = 1.5$ $y = 0.5$ we get two states with significant $B(M1)$ strength. The energies are 8.45 MeV and 9.26 MeV, and the values of $B(M1) \uparrow$ are 0.77 μ_N^2 and 2.70 μ_N^2. The orbital content is still negligible.

In Table VI we list for ^{10}Be those states which contain most of the orbital strength. There are both $T = 1$ and $T = 2$ states, and for $x = 1.5$ $y = 0.5$ they range in energy from about 16 MeV to 24 MeV. The orbital strength is at a much higher energy than the spin strength for ^{10}Be.

C. ^{12}C

As shown in Table VIII, there are two well-studied states in ^{12}C: the 1^+ $T = 0$ state at 12.71 MeV and the 1^+ $T = 1$ (isovector) state at 15.11 MeV. As mentioned in the introduction, the ratio $(|\frac{g_s(vector)}{g_s(scalar)})^2 = 81$ so we expect the 1^+ $T = 1$ state to be excited much more strongly than $T = 0$ and indeed in the calculation this is the case. For $x = 1$ $y = 1$, we get

$$\frac{B(M1) \uparrow (g.s. \rightarrow 1^+ \ T = 1)}{B(M1) \uparrow (g.s. \rightarrow 1^+ \ T = 0)} = \frac{0.856}{0.0026} = 330$$

The lowest calculated 1^+, $T = 1$ state in ^{12}C is dominantly a spin state. For $x = 1$ $y = 1$ the values of $B(M1)$ (in μ_N^2) are $B_l = 0.0314$, $B_\sigma = 0.584$ and $B(M1) = 0.886$. For the other extreme $(x = 1.5$ $y = 0.5)$, we obtain

$B_l = 6.91 \ 10^{-5}$, $B_\sigma = 2.571$ and $B(M1) = 2.54$. This analysis confirms the assumption made in testing the CVC theory by A. Richter at. al. -that the the 15.1 MeV state is indeed an almost pure spin state. Only then can a relation between the isovector $M1$ and a corresponding Gamow-Teller matrix element from beta decay be made.

Where there are orbital (scissors mode) excitations, the analysis is tricky. For $x = 1.5$ $y = 0.5$, the next excited $J = 1^+$ $T = 1$ state is at 17.46 MeV, and it has $B_l = 0.28 \ \mu_N^2$ and $B_\sigma = 0.074 \ \mu_N^2$. But there is a large cancellation between spin and orbit so that $B(M1)$ is only 0.068 μ_N^2. One would need in addition to electromagnetic excitation another probe e.g. proton scattering to unearth the substantial orbital strength at 20.19 MeV ($B_l = 0.17 \ \mu_N^2$, $B_\sigma = 0.0052 \ \mu_N^2$, $B(M1) = 0.113 \ \mu_N^2$).

We now consider the (x, y) dependence of $B(M1)$. For the four values (1,1), (1.5,1), (1,0.5) and (1.5,0.5), the values of $B(M1)$ (in μ_N^2) are 0.89, 1.89, 1.12 and 2.54 respectively. The experimental value is 2.63(8) μ_N^2 so we see that we need the full machinery (larger spin-orbit and weaker tensor interactions) to come even close to the large measured value. Again, as in ^8Be, the energies of the 1^+ states are somewhat too low, and changing x and y does not solve the problem. Thus for $x = 1.5$ $y = 0.5$ the 1^+ states are calculated to be at 11.8 MeV ($T = 0$) and 13.1 MeV ($T = 1$).

A comparison of experiment and theory for $B(E2) \uparrow$ in ^{12}C is made in Table IX. By far the largest $B(E2)$ is to the first 2^+ state at 4.44 MeV. With the 'standard' effective charges $e_p = 1.5$ $e_n = 0.5$, the calculated values of $B(E2)$ range from 70 to 80 e^2fm^4 -they are not very sensitive to variations in x and y. The experimental value is 40 e^2fm^4. It appears from the results here and in ^8Be that the effective charges in the $0p$ shell should be somewhat smaller than the standard value. For ^{12}C, and for all even-even nuclei considered here, the $B(E2)$ $T = 0 \rightarrow T = 0$ is proportional to $(e_p + e_n)^2$ and the $B(E2)$ $T = 0 \rightarrow T = 1$ to $(e_p - e_n)^2$.

D. ^{20}Ne

The experimental data for ^{20}Ne is contained in Tables X and XI. In ^{20}Ne there is a strong $M1$ excitation to a $J = 1^+$ $T = 1$ state at 11.26 MeV. The experimental values to this state (in μ_N^2) are $B_l = 0.52(15)$, $B_\sigma = 0.49(6)$ and $B(M1) = 2.02(36)$. The calculated values of $B(M1)$ for $x = 1$ $y = 1$ are $B_l = 0.43$, $B_\sigma = 0.06$ and $B(M1) = 0.83$. For $x = 1.5$ $y = 0.5$, we get $B_l = 0.47$, $B_\sigma = 0.35$ and $B(M1) = 1.63$. We thus get substantially better values of B_σ and hence of $B(M1)$ when we increase the spin-orbit interaction and decrease the tensor.

But, as mentioned previously, we appear to get the 'correct' spin-orbit splitting in ^{17}O with a value of the spin-orbit strength $x = 1$. This gives a $d_{3/2} - d_{5/2}$ single-particle splitting of 5.2 MeV. For $x = 1.5$ the value would be $5.2 \times 1.5 = 7.8 \ MeV$. Thus the $M1$'s in ^{20}Ne (and in other $s - d$ shell nuclei as well) act as if the spin-orbit splitting is much larger than what is obtained from energy splittings for $A = 17$.

When we further move the $1s$ level up by 2 MeV ($\Delta\epsilon_s = 2MeV$) relative to $0d$, the values of B_l, B_σ and $B(M1)$ to the strongest state (calculated to be at 9.76 MeV) become (in units of μ_N^2) 0.51, 0.62 and 2.26 respectively. We see that B_σ and hence $B(M1)$ increase and indeed, for $B(M1)$, we are within the errors bars of the experiment.

The dominant $E2$ strength goes to the 2_1^+ state at 1.63 MeV. The measured $B(E2) \uparrow$ is 340(22) e^2fm^4 whilst the calculated values (with $e_p = 1.5$, $e_n = 0.5$) are about 300 e^2fm^4 -in good agreement. As in ^{12}C, there is very little sensitivity of $B(E2)$ to x and y. This could be understood if the extreme LS coupling limit was valid. If the ground state is $L = 0$ $S = 0$ and the 2^+ state $L = 2$ $S = 0$ then the tensor and spin-orbit interactions would have no effect -the expectation values will vanish for $S = 0$ states. With $\Delta\epsilon_s = 2MeV$, the $B(E2)$ to the 2_1^+ state decreases to 265 e^2fm^4. There is some loss of collectivity when the $1s$ level is raised in energy.

E. ^{22}Ne

Experimentally (see Table XII), the $B(M1)$'s in ^{22}Ne are as follows: 0.43(17) μ_N^2 to a state at 5.33 MeV, 1.36(56) μ_N^2 to a state at 6.85 MeV and 1.80(7) to a state at 9.18 MeV. The summed experimental strength to the three states is 3.59 μ_N^2. For the four (x,y) values we get the sum to be 1.62 μ_N^2, 2.76 μ_N^2, 1.96 μ_N^2 and 3.38 μ_N^2. We clearly get better results when $x = 1.5$ (second and fourth cases), and we get the best results with $x = 1.5$, $y = 0.5$. In a later section, we shall see that even better results are obtained by increasing the $1s$ level ($\Delta\epsilon_s = 2MeV$).

In the absence of a spin-orbit interaction, the nucleus ^{22}Ne is triaxial in a Hartree-Fock calculation e.g. with a Skyrme interaction. Introducing a spin-orbit interaction tends to make this and all other nuclei axially symmetric. Depending on what parameters are used, the calculation can go either way -triaxial or axially symmetric.

We can determine the degree of triaxiality by looking at $B(E2) \uparrow$ from the ground state. In the axial case, the lowest 2^+ state will be a member of a $K = 0$ ground state band and the next one of a different band with $K = 2$. Thus $B(E2)$ $0_1 \to 2_1$ will be much stronger than $B(E2)$ $0_1 \to 2_2$. In the triaxial case, there will be much K mixing and we can get strong $B(E2)$'s to both the 2_1 and 2_2 states.

Experimentally, as shown in Table XIII, the $B(E2)$ to the first 2^+ state is *much* larger than to the second. The respective values are 300 e^2fm^4 and 12 e^2fm^4. These results favor axial symmetry for ^{22}Ne.

In the shell model calculations with the (x,y) interaction, we also see a strong dependence of the ratio $B(E2)_2$ to $B(E2)_1$ with the strength of the spin-orbit interaction, and indeed the results are consistent with those of Hartree-Fock calculations.

For $x = 1$ (i.e. the free-space spin-orbit interaction) and $y = 1$, we get strong $B(E2)$'s to both the first and second 2^+ states. The respective values are 199 e^2fm^4 and 113 e^2fm^4. When we increase x to 1.5 (keeping $y = 1$) the second $B(E2)$ shrinks considerably. The values are now $B(E2)_1$=289 e^2fm^4 and $B(E2)_2$= 7.5 e^2fm^4.

Thus the $B(E2)$ data, just like the $M1$ data, favour a larger spin-orbit interaction than in free space. Thus we have the interesting observation that the absence of triaxiality throughout the periodic table may be due to the fact that the spin-orbit interaction in a nucleus gets enhanced by medium modifications. As mentioned in the introduction, there are several models for such a modification -a Dirac effective mass in the Walecka model, and three-body interactions due to meson degrees of freedom in the Argonne-Illinois model.

F. ^{24}Mg

In ^{24}Mg we have the (e,e') results of Richter et. al. [3] as shown in Table XIV. They obtain a summed $B(M1)$ of 4.85(36) μ_N^2 up to a maximum excitation energy $E_{max} = 11.4$ MeV, and the sum increases to 5.84(40) μ_N^2 for $E_{max} = 15$ MeV.

In our calculation the summed $B(M1)$ strength to *all* states in the sd shell for the four values of (x,y) are 3.78, 4.95, 3.87 and 5.43 μ_N^2. The sums for the first 10 states (which have maximum excitation energies which vary from 15 to 16 MeV i.e. in the same ballpark as the experiment) are smaller: 2.80, 3.28, 2.90 and 3.45 μ_N^2. We thus find that the experimental sum is larger than the theoretical sum by a factor $\frac{exp}{free} = \frac{5.43}{3.45} = 1.57$. The USD interaction of Wildenthal also gives a ratio bigger than one but it is much smaller than ours: 1.11(8).

However, with $\Delta\epsilon_s = 2MeV$ in the $x = 1.5$ $y = 0.5$ case, the situation improves a great deal. The total sum increases to 6.44 μ_N^2, and the sum to the first ten states increases to 4.58 μ_N^2.

Experimentally, two low-lying states absorb most of the $M1$ strength. We have $B(M1) \uparrow= 1.5\mu_N^2$ to a 9.96 MeV state and 2.5 μ_N^2 to a 10.71 MeV state. In the calculation however, for say (x,y)=(1.5,0.5), the values of $B(M1)$ greater than 0.3 μ_N^2 are 2.11 μ_N^2 to a 9.32 MeV state and 0.64 μ_N^2 to a 9.47 MeV state. Since in the shell model

calculation these two states are nearly degenerate, it is hard to get the strengths just right. The experimental summed strength to the two states is 4 μ_N^2 whilst the calculation gives 2.75 μ_N^2.

In ^{24}Mg we also have results for B_σ from proton inelastic scattering by Crawley $et.$ $al.$ [18] in (p,p') reactions as shown in Table XV (see also Table XVI). The summed experimental strength for states ranging in energy from 8.865 to 14.267 MeV is 5.86 μ_N^2. The first two 1^+ states at 8.865 and 9.820 are weakly excited: $B_\sigma = 0.042 \pm 0.011$ and 0.260 ± 0.040 μ_N^2 respectively. The next two states at 9.962 and 10.711 MeV are strongly excited: $B_\sigma = 1.160 \pm 0.180$ and 3.180 ± 0.300 μ_N^2 respectively. The summed experimental strength for the above-mentioned energy range is 5.862 μ_N^2.

Theoretically, the entire summed strength of B_σ for the four values of (x, y) are respectively 1.73, 3.35, 1.94 and 4.04 μ_N^2. For the first ten states we get (for $x = 1.5$, $y = 0.5$) $\sum B_\sigma = 1.79\mu_N^2$. This is considerably smaller than the experimental sum. But with $\Delta\epsilon_s = 2MeV$ the total B_σ sum is 5.45 μ_N^2 and to the first ten states 3.38 μ_N^2.

As to individual levels and for $x = 1.5$ $y = 0.5$, we get nearly degenerate states at 9.325 and 9.466 MeV with $B_\sigma = 1.02$ and 0.27 μ_N^2 i.e. a summed strength of 1.29 μ_N^2. The summed experimental strength $\sum B_\sigma$ for the two strong states is 4.34 μ_N^2. With $\Delta\epsilon_s = 2MeV$ the corresponding summed B_σ strength is 2.41 μ_N^2, somewhat closer to experiment.

G. ^{28}Si

From the work of Lüttge $et.$ $al.$ [19] as shown in Table XVIII, the summed $B(M1) \uparrow$ strength in ^{28}Si up to an excitation energy of 18 MeV is 7.53 μ_N^2. There is considerable fragmentation with the three strongest states at energies of 10.901, 11.445 and 12.331 MeV having $B(M1)$'s of $0.90 \pm 0.02\mu_N^2$, $4.42 \pm 0.20\mu_N^2$ and $0.87 \pm 0.06\mu_N^2$ respectively. From the work of Crawley $et.$ $al.$ (Table XIX), the summed B_σ strength up to an excitation energy of 15.8 MeV is 9.51 μ_N^2. The B_σ strength to the strongest state at 11.45 MeV is 3.32 ± 0.24 μ_N^2 as determined by Crawley $et.$ $al.$ [18].

For $x = 1$ $y = 1$, our summed $B(M1)_{isovector}$ is 4.44 μ_N^2 with $\sum B_l = 2.24\mu_N^2$ and $\sum B_\sigma = 2.39\mu_N^2$. Our result is significantly less than experiment. The experimental $B(M1)$ sum to the first ten states is 7.35 μ_N^2 whereas our sum is only 2.93 μ_N^2. The calculated $B(M1)$ strength to individual states for $x = 1$ $y = 1$ are 1.35, 0.49, 0.39 and 0.35 μ_N^2 at 10.31, 11.78, 12.97 and 14.54 MeV respectively. We seem to be missing the very strong state with an experimental $B(M1)$ of 4.42 μ_N^2.

The results are dramatically improved when we go to $x = 1.5$. The value of $B(M1)$ to the single strongest state for the cases (x, y)=(1,1) and (1.5,0.5) are respectively 1.35 μ_N^2 and 3.90 μ_N^2. There is almost a factor of three increase and we come close to the experimental value of 4.42 μ_N^2. The corresponding change in B_σ shown in Tables XIX, XX and XXI is from 0.86 μ_N^2 to 2.26 μ_N^2, in much better agreement but still lower than the experimental value. From (x, y)=(1,1) to (1.5,0.5) the *total summed strength* (Table II) for $B(M1)$ changes from 4.44 to 6.38 μ_N^2, whilst the total summed B_σ strength changes from 2.39 to 6.79 μ_N^2. If we limit the sum to the first ten states (Table III), the change in B_σ is from 1.29 to 4.76 μ_N^2. In the latter case, B_σ increases by a factor of 3.7.

We nest consider the case (x, y)=(1.5,0.5) with $\Delta\epsilon_s = 2MeV$. The values of $B(M1)$ and B_σ to the single strongest state (calculated to be at 10.03 MeV) are 4.61 μ_N^2 and 4.26 μ_N^2 respectively. The total summed strength for $B(M1)$ and B_σ are now increased to 8.94 and 9.04 μ_N^2, whilst to the first ten states the sums are 7.08 and 7.24 μ_N^2.

In contrast to B_σ, there is a substantial decrease in B_l when we go from (x, y)=(1,1) to (1.5,0.5). The respective values for the total B_l strengths are 2.24 μ_N^2 and 1.46 μ_N^2 respectively. With (x, y)=(1.5,0.5) and $\Delta\epsilon_s = 2MeV$, there is a further decrease to 1.36 μ_N^2.

H. ^{32}S

The results are shown in Table XXII. The most clearcut data in ^{32}S appears for B_σ by Crawley *et. al.* [18]. The summed strength is 10.75 μ_N^2 and the strongest excited states and the corresponding (B_σ in μ_N^2) are: 8.13 MeV (1.46 $\pm$ 0.19), 11.13 MeV (4.08 $\pm$ 0.53) and 11.63 MeV (2.38 $\pm$ 0.35). For $x = 1.5$ $y = 0.5$ our summed B_σ strength from table XXIV is 11.16 μ_N^2 to the first ten states and 12.03 μ_N^2 to all states. The agreement with experiment is reasonable. Our strongest state, however, has a B_σ strength of 5.98 μ_N^2 at 11.21 MeV.

When for the case $x = 1.5$ $y = 0.5$ case we also use $\Delta\epsilon_s = 2MeV$, the value of B_σ to the first ten states drops somewhat to 9.45 μ_N^2, and the total sum also drops to 10.91 μ_N^2. The strongest state calculated to be at 10.25 MeV now has $B_\sigma = 4.20\mu_N^2$, a significant drop from 5.98 μ_N^2.

Note that this is the first time we get a drop in B_σ when $\Delta\epsilon_s = 2MeV$ is used; for ^{20}Ne, ^{22}Ne, ^{24}Mg and ^{28}Si there was a rise. Note further that our results with this drop are in better agreement with experiment.

The $B(M1)$ data is a bit complicated. In Table XXII from the results reported in the paper of Crawley *et. al.* [18] only a few states are seen, with a summed strength of 5.43 μ_N^2. However, there is other data by Petraitis *et. al.* [20] which shows strength to many other states. The summed Peraitis strength is 4.21 μ_N^2. However, he does not have the state at 8.13 MeV ($B(M1) \uparrow = 1.14 \pm 0.18$ μ_N^2), and disagrees on the $B(M1)$ to the 11.13 MeV state for which Petraitis gets 1.24 ± 0.13 μ_N^2 but ref. [18] reports 2.40 ± 0.22 μ_N^2.

We feel the best estimate at the moment is to take all the $M1$ states of ref. [18] and add to that the strength of Petraitis *et. al.* to states not found in ref. [18]. Thus we get schematically strength reported in ref. [18] plus (strength reported by Petraitis *et. al.* minus strength to states at 9.66 MeV, 11.16 MeV and 11.85 MeV). When all this is done, we get a summed $B(M1)$ strength of 7.61 μ_N^2. Our calculated $B(M1)$ strength (with $x = 1.5$ $y = 0.5$) is 10.98 μ_N^2 to all states. With $\Delta\epsilon_s = 2MeV$, this *drops* to 10.12 μ_N^2, whilst the summed strength to the first ten states (i.e. up to 15.1 MeV excitation energy) is 8.81 μ_N^2.

It is difficult to draw a definitive conclusion for ^{32}S because two sets of $M1$ data are not in accord. From the B_σ data, it appears that we need an enlarged spin-orbit interaction. For $x = 1$ $y = 1$ the sum B_σ is only 2.73 μ_N^2 whereas for $x = 1.5$ $y = 0.5$ it is 12.03 μ_N^2, closer to the experimental sum of 10.75 μ_N^2. However, the $M1$ data supports a somewhat weaker spin-orbit and/or a stronger tensor interaction. Again, from Table II, the case $x = 1.5$ $y = 1$ gives results of 8.69 μ_N^2 for $B(M1)$ and 8.52 μ_N^2 for B_σ.

It is significant that raising the $1s$ level relative to $0d$ causes a rise in B_σ for ^{20}Ne, ^{24}Mg and ^{28}Si, whilst for ^{32}S and as we shall see ^{36}Ar it causes a decrease in B_σ.

I. ^{36}Ar

For ^{36}Ar, we refer to Tables XXVII, XXVIII and XXIX. Because of the large error bars on the (e, e') experiment, we will concern ourselves here with the summed strength only. The measured value is 2.65 ± 0.1 μ_N^2.

The total summed strengths for the four values of (x, y) are 3.35, 5.19, 3.72, 5.71 μ_N^2 respectively. The sums to the first ten states are 2.98, 4.82, 3.58 and 5.50 μ_N^2 respectively. They are almost the same as the complete sums.

The Foltz work [21] contains a theoretical discussion of shell model calculations using the Wildenthal interaction [4]. Even with this phenomenological interaction there is a difficulty in the sense that the theory predicts a strong excitation of $B(M1) = 1.3$ μ_N^2 at 10.3 MeV, whereas experimentally there is very strong fragmentation. Also the theory with bare operators predicts too large a summed strength (5.13 μ_N^2) for all the strength, and 4.73 μ_N^2 for the strength below 15 MeV. However, using effective operators, whose values are consistent with the fundamental calculations of Towner and Khanna [22], the summed strength is reduced and is closer to experiment. The above two numbers now become 3.54 μ_N^2 to all states and 2.86 μ_N^2 to states below 15 MeV.

Thus the results we obtain with $x = 1.5$ $y = 0.5$, although still too large, are not so different from those obtained in the theoretical discussion of Foltz et. al. [21]. As seen in Tables III and IV, we get summed strengths of 5.71 μ_N^2 to all states and 5.50 μ_N^2 to the first ten states. With $\Delta\epsilon_s = 2MeV$, the corresponding numbers are 5.48 and 5.10 μ_N^2.

J. ^{44}Ti

There are no $M1$ data for ^{44}Ti so we concentrate on $B(E2)$'s -these are shown in Tables XXX and XXXI. The experimental value of $B(E2) \uparrow$ is 540(140) $e^2 fm^4$ but the calculated values (with $e_p = 1.5$ $e_n = 0.5$) are much larger -from 908 to 925 $e^2 fm^4$ for various combinations of x and y.

The probable reason for the calculated over-collectivity is that the $1p$ level is too low in energy relative to $0f$. This is confirmed in calculations in which we raise the $1p$ centroid by 2 MeV relative to the $0f$ centroid. In that case, the value of $B(E2) \uparrow$ is reduced to 710 $e^2 fm^4$ (for $x = 1.5$ $y = 0.5$).

VII. CLOSING REMARKS

In this work we simulate a realistic $G-$matrix by the interaction $V_c(r) + xV_{s.o} + yV_t$ where $x = 1$, $y = 1$ gives us matrix elements close to those obtained in a non-relativistic approach. We must emphasize at first that we use the same interaction to calculate single-particle energies as is used for the valence interactions. In this, we differ from most other calculations in which a hybrid approach is taken -most often the single-particle energies are taken from experiment. We feel however that one is not really testing the interaction. Later we relax this condition somewhat by shifting the $1s$ single-particle energy relative to $0d$.

In an open-shell nucleus the tensor interaction has the opposite effect of the spin-orbit interaction [12]. That is to say when the strength of the tensor interaction is *decreased*, the summed spin strength $\sum B_\sigma$ *increases*. There seems to be a phenomenological need for a weaker tensor interaction in a nucleus than in free space, and several arguments have been proposed to justify this. At the simplest level, there is the 'self-weakening mechanism' [10,24]. This simply means that the effects of higher-shell admixtures (i.e. $\Delta N = 2$ excitations and beyond) can be simulated in the valence space by using a weaker tensor interaction. This is a purely nuclear structure justification, and the calculations for these admixtures can be carried out either perturbatively or in large shell model spaces. There is also the 'universal scaling' idea of Brown and Rho [9] that all mesons, except for the pion, are less massive inside a nuclear medium than in free space. This leads to a larger range for the contribution to the tensor interaction of ρ-exchange. This then causes a greater cancellation between the ρ- and π-exchange contributions to the tensor interaction, leading to the overall weakening of the tensor interaction.

In our calculations, B_σ does indeed increase when the tensor interaction is cut in half. Although the effect is not so large as increasing the spin-orbit interaction (from $x = 1$ to $x = 1.5$), overall the effects seem beneficial with the possible exception of ^{36}Ar.

It should be further noted that the tensor interaction seems to have very little effect on the $B(E2)$ $0_1 \rightarrow 2_1$. This is undoubtedly because for these states one is approaching the LS limit, and the wavefunctions are dominantly $S = 0$. The tensor interaction vanishes for $S = 0$ states. To our mind, this means that one can control B_σ to some extent without affecting the low-lying spectroscopy. Note that for the $N = Z$ nuclei considered here, the isovector transition B_σ $(J = 0$ $T = 0$ $\rightarrow J = 1 T = 1)$ is proportional to the Gamow-Teller transition $B(GT)$. It has often been said that $B(GT)$ has to be quenched and there are even more specific statements that the GT coupling constant which is 1.251 in free space should be equal to unity inside the nucleus. Our calculations here cast some doubt on such detailed conclusions. First of all, the famous sum rule $B(GT)_+ = 3(N - Z) + B(GT)_-$ puts no restrictions whatsoever on $B(GT)$ for an $N = Z$ nucleus- one only gets the result $B(GT)_+ = B(GT)_-$ which is simply isospin conservation.

Secondly, we know that B_σ isovector and $B(GT)$ will vanish in the $SU(4)$ limit for $N = Z$ nuclei. At the other limit of jj coupling, B_σ and $B(GT)$ are much too large. So it seems we can get almost any answer we want. Indeed, for $x = 1$ $y = 1$ we get too small a $B(GT)$ and if we really believed in this interaction we could conclude that $B(GT)$ had to be *enhanced*. On the other hand, if we had chosen $x = 2$ $y = 1$ (a very large spin-orbit interaction), we could conclude that $B(GT)$ had to be quenched. Perhaps a case can be made that the $3(N - Z)$ part of the sum rule is quenched but not the $B(GT)$ part.

On the other hand, in the work of Richter, Weiss, Hausser and Brown [3], the discussion of quenching pertains to the ratio of $B(M1)_\sigma$ to $B(GT)$, and here one is on much firmer ground. By taking the ratio the uncertainties of the details of the nuclear interaction largely disappear, and the conclusion that $B(GT)$ is more quenched than $B(M1)$ is certainly valid.

In the $0p$ shell, where there is only one l orbit, it is easier to draw conclusions. With $x = 1$ $y = 1$, we get a $B(M1)$ of only 0.89 μ_N^2 to the 15.11 MeV state in ^{12}C-the experimental value is 2.63(8) μ_N^2. For $x = 1.5$ $y = 1$, the value is 1.89 μ_N^2, but only when we both increase the spin-orbit and decrease the tensor interaction ($x = 1.5$ $y = 0.5$) do we get a $B(M1)$ comparable with experiment (2.55 μ_N^2).

In the $s - d$ shell, things are somewhat more complicated by the fact that there are two l orbits. Our initial calculation (in which the $0d_{5/2}$, $1s_{1/2}$ and $0d_{3/2}$ single-particle energies are calculated with the same (x, y) interaction as is used in the valence space) puts the centroid of the $1s$ level too low in energy relative to that of $0d$. This has the effect of making B_σ too small in the lower half of the $s - d$ shell e.g. ^{20}Ne, ^{22}Ne, ^{24}Mg and ^{28}Si. By moving the $1s$ level up by 2 MeV we increase B_σ significantly for these nuclei, and as a bonus, in the upper half of the $s - d$ shell i.e. ^{32}S and ^{36}Ar B_σ gets slightly decreased, and this appears all to the good.

We find that for most nuclei, the non-relativistic $G-$matrix, as given by the parameters $x = 1$ $y = 1$ gives much too small values for the spin excitation rates (B_σ). By increasing the two-body spin-orbit term from $x = 1$ to $x = 1.5$ we can greatly improve the situation. Microscopic justification for this came from the Dirac phenomenology in which a Dirac effective mass m^* is introduced, and is said to arise from the exchange of σ mesons between nucleons. The spin-orbit interaction gets increased by a factor $\frac{m}{m^*}$ (however m^* is a function of r and so the effect should be greater in the interior than on the surface of the nucleus). There is an alternate three-body approach [7] which also gives an enhanced spin-orbit splitting.

Lastly, we have here introduced a new topic -the cross-correlation between the spin and orbital parts of $B(M1)$. In our calculations, we find that the the total sum $\sum B(M1)$ is very close to $\sum B_\sigma + \sum B_l$, indicating that the sum of the cross terms nearly vanishes.

VIII. ACKNOWLEDGEMENTS

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TABLE I. Single-Particle Energies Calculated with the (x, y) interaction $(x = 1, \; y = 1)$ in a small space $(0\hbar\omega)$ and a large space $(0 + 2)\hbar\omega$.

Nucleus	State	$0\hbar\omega$	$(0 + 2)\hbar\omega$
^5He	$3/2^-$	0	0
	$1/2^-$	3.375	2.224
^{15}O	$1/2^-$	0	0
	$3/2^-$	5.063	5.679
^{17}O	$1/2^+$	0	0
	$5/2^+$	0.119	1.430
	$3/2^+$	5.681	7.092
^{39}Ca	$3/2^+$	0	
	$1/2^+$	5.457	
	$5/2^+$	6.973	
^{41}Ca	$3/2^-$	0	
	$7/2^-$	0.940	
	$1/2^-$	2.458	
	$5/2^-$	7.966	

TABLE II. The Summed isovector $M1$ strength in μ_N^2 for various strengths (x, y) of the spin-orbit and tensor interaction

Nucleus	x	y	$B(M1)$	B_l	B_σ
^8Be	1	1	1.05	0.67	0.38
	1.5	1	1.28	0.64	0.65
	1	0.5	1.06	0.67	0.41
	1.5	0.5	1.33	0.64	0.73
^{10}Be	1	1	2.09	0.111(0.025)	1.62
$T = 1 \rightarrow T = 1$	1.5	1	3.17	0.105(0.036)	2.64
	1	0.5	2.46	0.111(0.028)	2.03
	1.5	0.5	3.55	0.103(0.043)	3.11
^{10}Be	1	1	0.060	0.151	0.093
$T = 1 \rightarrow T = 2$	1.5	1	0.029	0.144	0.189
	1	0.5	0.046	0.152	0.104
	1.5	0.5	0.050	0.142	0.246
^{12}C	1	1	1.42	0.60	0.91
	1.5	1	2.29	0.50	2.07
	1	0.5	1.58	0.59	1.13
	1.5	0.5	2.85	0.46	2.82
^{20}Ne	1	1	1.53	0.95	0.59
	1.5	1	2.34	1.09	1.28
	1	0.5	1.57	0.96	0.63
	1.5	0.5	2.54	1.10	1.49
$\Delta\epsilon_s = 2MeV$[a]	1.5	0.5	3.50	1.39	2.21
^{22}Ne	1	1	2.28	0.029 (0.36)[b]	1.81
$T = 1 \rightarrow T = 1$	1.5	1	3.98	0.046(0.44)	3.20
	1	0.5	2.51	0.030 (0.37)	2.02
	1.5	0.5	4.51	0.054 (0.46)	3.69
$\Delta\epsilon_s = 2MeV$	1.5	0.5	5.79	0.068 (0.486)	4.32
^{22}Ne	1	1	0.34	0.27	0.16
$T = 1 \rightarrow T = 2$	1.5	1	0.38	0.31	0.39
	1	0.5	0.32	0.28	0.17
	1.5	0.5	0.41	0.32	0.51
$\Delta\epsilon_s = 2MeV$	1.5	0.5	0.48	0.38	0.76
^{24}Mg	1	1	3.78	2.08	1.73
	1.5	1	4.95	1.75	3.35
	1	0.5	3.87	2.02	1.94
	1.5	0.5	5.43	1.69	4.04
$\Delta\epsilon_s = 2MeV$	1.5	0.5	6.45	1.63	5.46
^{28}Si[c]	1	1	4.44	2.24	2.39
	1.5	1	6.38	1.60	5.33
	1	0.5	4.59	2.06	2.79
	1.5	0.5	7.54	1.46	6.97

$\Delta\epsilon_s = 2MeV$	1.5	0.5	8.94	1.36	9.04
^{32}S	1	1	4.76	2.32	2.73
	1.5	1	8.69	1.57	8.52
	1	0.5	5.43	2.17	3.76
	1.5	0.5	10.96	1.15	12.03
$\Delta\epsilon_s = 2MeV$	1.5	0.5	10.19	1.24	2.21
^{36}Ar	1	1	3.35	1.44	2.25
	1.5	1	5.19	0.95	5.20
	1	0.5	3.72	1.33	2.87
	1.5	0.5	5.71	0.87	6.05
$\Delta\epsilon_s = 2MeV$	1.5	0.5	5.48	0.90	5.72
^{44}Ti	1	1	2.41	1.50	0.89
	1.5	1	3.41	1.63	1.79
	1	0.5	2.42	1.47	0.94
	1.5	0.5	3.64	1.64	2.04
$\Delta\epsilon_p = 2MeV$[d]	1.5	0.5	5.56	2.13	3.53

[a]The $1s$ orbital is raised by 2 MeV above the $0d$ orbital

[b]The first number in this column is the *isoscalar* summed strength; the second number (in parentheses) is the *isovector* summed strength.

[c]For (x, y)=(1,1) the sum is over the 500 lowest states, but for all the other (x, y) pairs the sum is over the lowest 50 states only.

[d]The $1p$ orbital is raised by 2 MeV above the $0f$ orbital

TABLE III. Summed isovector $M1$ strength (First Ten States and All States) for $x = 1.5$, $y = 0.5$

^{20}Ne	$B(M1)$	B_l	B_σ
1st 10 States	2.05	0.90	1.22
All States	2.54	1.10	1.49
$^{22}Ne\ T = 1\ \rightarrow T = 1$			
1st 10 States	3.71	0.23	2.88
All States	4.51	0.46	3.69
$^{22}Ne\ T = 1\ \rightarrow T = 2$			
1st 10 States	0.23	0.25	0.36
All States	0.41	0.31	0.51
^{24}Mg			
1st 10 States	3.45	0.94	1.79
All States	5.43	1.68	4.03
^{28}Si			
1st 10 States	5.30	0.66	4.76
All States	7.54	1.46	6.97
^{32}S			
1st 10 States	10.13	0.53	11.16
All States	10.98	1.15	12.03
^{36}Ar			
1st 10 States	5.50	0.63	5.95
All States	5.71	0.87	6.05
^{44}Ti			
1st 10 States	2.73	1.18	1.10
All States	3.64	1.64	2.04

TABLE IV. Same as Table III ($x = 1.5$, $y = 0.5$) but with $\Delta\epsilon_s = 2MeV$ for the $s - d$ shell nuclei and $\Delta\epsilon_p = 2MeV$ for ^{44}Ti

^{20}Ne	$B(M1)$	B_l	B_σ
1st 10 States	3.39	1.25	2.09
All States	3.50	1.39	2.21
$^{22}Ne\ T = 1\ \rightarrow T = 1$			
1st 10 States	5.20	0.25	4.18
All States	5.79	0.49	4.80
$^{22}Ne\ T = 1\ \rightarrow T = 2$			
1st 10 States	0.36	0.29	0.67
All States	0.48	0.38	0.76
^{24}Mg			
1st 10 States	4.59	0.81	3.38
All States	6.45	1.63	5.45
^{28}Si			
1st 10 States	7.08	0.52	7.24
All States	8.94	1.36	9.04
^{32}S			
1st 10 States	8.81	0.41	9.45
All States	10.19	1.24	10.91
^{36}Ar			
1st 10 States	5.10	0.48	5.21
All States	5.48	0.90	5.72
^{44}Ti			
1st 10 States	3.73	1.32	1.26
All States	5.56	2.13	3.52

TABLE V. B_l, B_σ and $B(M1)$ strengths in μ_N^2 for states with $B(M1)$ (usually) $\geq 0.3\mu_N^2$ as a function of x and y -the strengths of the spin-orbit and tensor interactions respectively

Nucleus	x	y	$E_x(MeV)$	B_l	B_σ	$B(M1)$
^8Be	1	1	13.72	0.257	0.115	0.716
	1.5	1	13.06	0.267	0.218	0.967
	1	0.5	13.75	0.255	0.139	0.770
	1.5	0.5	13.05	0.258	0.249	1.014
^{10}Be	1	1	7.68	0.017	1.512	1.849
	1.5	1	8.84	0.000	2.837	2.837
	1	0.5	7.71	0.001	2.333	1.966
	1.5	0.5	8.45	0.004	0.856	0.774
			9.26	0.000	2.232	2.695
^{12}C	1	1	13.60	0.031	0.583	0.886
	1.5	1	13.08	0.004	1.710	1.887
	1	0.5	13.33	0.012	0.901	1.121
	1.5	0.5	13.11	0.000	2.571	2.545
^{20}Ne	1	1	12.40	0.434	0.064	0.830
			19.21	0.136	0.047	0.344
	1.5	1	10.73	0.470	0.288	1.494
	1	0.5	12.18	0.240	0.003	0.187
			12.25	0.238	0.136	0.735
	1.5	0.5	10.66	0.475	0.346	1.631
$\Delta\epsilon_s = 2MeV$	1.5	0.5	9.757	0.511	0.623	2.263
			16.91	0.037	0.532	0.307
^{22}Ne	1	1	8.45	0.005	1.121	1.464
	1.5	1	7.06	0.176	0.053	0.461
			8.10	0.015	0.875	0.944
			10.00	0.006	0.370	0.374
			10.61	0.012	1.086	1.359
	1	0.5	8.63	0.001	1.492	1.800
	1.5	0.5	6.99	0.199	0.170	0.785
			8.57	0.011	1.077	1.065
			10.95	0.001	1.275	1.518
$\Delta\epsilon_s = 2MeV$	1.5	0.5	7.320	0.152	0.732	1.711
			8.575	0.009	1.664	1.544
			11.54	0.004	1.111	1.279
^{24}Mg	1	1	10.27	0.280	0.492	1.516
	1.5	1	9.35	0.204	0.943	2.024
			12.66	0.076	0.077	0.304
	1	0.5	10.17	0.280	0.595	1.693
	1.5	0.5	9.32	0.195	1.024	2.111
			9.466	0.083	0.265	0.644

$\Delta\epsilon_s = 2MeV$	1.5	0.5	9.401	0.183	2.316	3.799
			14.89	0.06	0.163	0.422
^{28}Si	1	1	10.31	0.055	0.860	1.352
			11.78	0.122	0.123	0.491
			12.97	0.359	0.001	0.387
			14.53	0.399	0.002	0.350
	1.5	1	9.64	0.075	2.810	3.801
	1	0.5	10.11	0.064	1.413	2.06
			14.54	0.383	0.001	0.352
	1.5	0.5	9.44	0.221	2.266	3.903
			10.54	0.062	1.313	0.804
$\Delta\epsilon_s = 2MeV$	1.5	0.5	10.03	0.007	4.258	4.612
			10.99	0.005	0.486	0.395
			11.64	0.028	1.073	0.756
			12.71	0.136	0.252	0.759
^{32}S	1	1	10.28	0.009	0.855	1.039
			11.17	0.111	0.266	0.721
			12.12	0.423	0.009	0.555
	1.5	1	9.63	0.007	4.574	4.212
			10.53	0.001	1.378	1.447
			11.77	0.163	0.242	0.801
	1	0.5	9.78	0.001	1.447	1.529
			11.14	0.097	0.753	1.391
	1.5	0.5	10.05	0.017	3.372	2.912
			11.21	0.018	5.976	5.341
			13.01	0.116	0.231	0.676
			13.81	0.105	0.979	0.442
^{36}Ar	1	1	10.70	0.002	1.180	1.290
			13.79	0.126	0.242	0.718
			15.50	0.237	0.005	0.311
	1.5	1	11.04	0.009	2.699	2.433
			12.10	0.053	0.105	0.307
			16.66	0.000	0.488	0.491
	1	0.5	10.97	0.016	1.639	1.981
			12.29	0.194	0.057	0.460
			16.41	0.071	0.174	0.466

	1.5	0.5	11.91	0.018	3.069	2.622
			12.97	0.041	1.010	1.457
			13.73	0.017	0.376	0.553
			17.65	0.002	0.431	0.488
$\Delta\epsilon_s = 2MeV$	1.5	0.5	10.78	0.073	0.817	0.402
			12.02	0.028	1.900	2.386
			12.30	0.025	0.519	0.771
			14.87	0.016	0.526	0.387
			15.16	0.027	0.509	0.300
			16.52	0.006	0.471	0.584
^{44}Ti	1	1	9.22	0.275	0.122	0.763
			11.91	0.527	0.015	0.365
	1.5	1	9.01	0.407	0.397	1.610
			9.50	0.356	0.007	0.465
	1	0.5	9.19	0.266	0.148	0.810
			12.76	0.020	0.205	0.352
			8.98	0.471	0.474	1.889
			9.49	0.303	0.001	0.340

TABLE VI. B_l, B_σ and $B(M1)$ strengths in μ_N^2 for states of interest not contained in Table X, and for $x = 1.5$ $y = 0.5$

Nucleus	$E_x(MeV)$	B_l	B_σ	$B(M1)$
^8Be	17.42	0.230	0.185	0.003
	19.51	0.141	0.023	0.050
^{10}Be	16.11	0.020	0.000	0.025
$T = 1$	19.50	0.026	0.001	0.011
	20.57	0.039	0.010	0.009
$T = 2$	18.74	0.089	0.217	0.028
	23.47	0.046	0.012	0.010
^{12}C	17.46	0.284	0.074	0.068
	20.18	0.167	0.005	0.112
^{20}Ne	11.50	0.136	0.108	0.001
	18.09	0.239	0.361	0.013
$\Delta\epsilon_s = 2MeV$	12.80	0.290	0.162	0.018
^{24}Mg	12.14	0.205	0.001	0.235
	13.92	0.237	0.384	0.017
	15.00	0.115	0.002	0.146
$\Delta\epsilon_s = 2MeV$	13.08	0.177	0.144	0.002
^{28}Si	13.26	0.142	0.381	0.058
$\Delta\epsilon_s = 2MeV$	10.79	0.104	0.003	0.140
	13.25	0.196	0.682	0.147
	14.47	0.003	0.214	0.168
^{32}S	17.01	0.140	0.021	0.270
$\Delta\epsilon_s = 2MeV$	10.98	0.149	0.525	0.114
	15.10	0.106	0.415	0.015
^{36}Ar	7.80	0.305	0.320	0.000
$\Delta\epsilon_s = 2MeV$	7.93	0.291	0.271	0.000
^{44}Ti	11.52	0.127	0.028	0.035
	12.77	0.095	0.469	0.142
$\Delta\epsilon_p = 2MeV$	9.93	0.323	0.130	0.043

TABLE VII. Isoscalar and Isovector $B(E2)$ (in e^2fm^4) as a function of the strengths (x, y) of the spin-orbit and tensor interaction in small space.

Nucleus	x	y	$B(E2)^a_{isoscalar}$	$B(E2)^b_{isovector}$
^8Be	1	1	71.27	9.26
	1.5	1	70.41	10.19
	1	0.5	71.24	9.35
	1.5	0.5	70.16	10.44
^{10}Be	1	1	73.18	29.83
$T = 1 \rightarrow T = 1$	1.5	1	71.49	27.90
	1	0.5	72.44	28.47
	1.5	0.5	70.29	27.38
^{10}Be	1	1	0	3.144
$T = 1 \rightarrow T = 2$	1.5	1	0	3.512
	1	0.5	0	3.164
	1.5	0.5	0	3.564
^{12}C	1	1	82.73	13.56
	1.5	1	78.43	17.72
	1	0.5	82.00	14.16
	1.5	0.5	75.30	19.20
^{20}Ne	1	1	315.1	53.44
	1.5	1	313.5	45.24
	1	0.5	316.0	52.68
	1.5	0.5	311.1	44.56
$\Delta\epsilon_s = 2MeV$	1.5	0.5	266.7	39.80
^{22}Ne	1	1	352.7	108.3
$T = 1 \rightarrow T = 1$	1.5	1	350.4	101.1
	1	0.5	353.8	107.9
	1.5	0.5	344.8	98.64
$\Delta\epsilon_s = 2MeV$	1.5	0.5	298.0	90.17
^{22}Ne	1	1	0	22.12
$T = 1 \rightarrow T = 2$	1.5	1	0	16.39
	1	0.5	0	21.26
	1.5	0.5	0	15.12
$\Delta\epsilon_s = 2MeV$	1.5	0.5	0	14.48
^{24}Mg	1	1	515.8	71.07
	1.5	1	511.3	78.89
	1	0.5	515.6	72.87
	1.5	0.5	508.9	80.68
$\Delta\epsilon_s = 2MeV$	1.5	0.5	481.8	82.86
$^{28}Si^c$	1	1	651.8	N/A

	1.5	1	574.0	N/A
	1	0.5	635.4	N/A
	1.5	0.5	546.9	26.4
$\Delta\epsilon_s = 2MeV$	1.5	0.5	439.5	N/A
^{32}S	1	1	529.8	80.41
	1.5	1	337.2	123.8
	1	0.5	488.5	89.66
	1.5	0.5	254.8	130.3
$\Delta\epsilon_s = 2MeV$	1.5	0.5	321.7	124.7
^{36}Ar	1	1	331.2	59.01
	1.5	1	289.0	83.00
	1	0.5	322.6	63.87
	1.5	0.5	280.9	85.71
$\Delta\epsilon_s = 2MeV$	1.5	0.5	307.9	75.91
^{44}Ti	1	1	963.9	119.8
	1.5	1	953.7	109.9
	1	0.5	967.1	120.3
	1.5	0.5	946.9	109.2
$\Delta\epsilon_p = 2MeV$	1.5	0.5		

(a) $e_p = 1.5$, $e_n = 0.5$

(b) $e_p = 1$, $e_n = -1$

(c) For $(x,y)=(1,1)$ the sum is over the 500 lowest states, but for all the other (x,y) pairs the sum is over the lowest 50 states only.

TABLE VIII. B(M1) strength in ^{12}C in 0 $\hbar\omega$ space (0 $\hbar\omega$).

	$1^+;0$ state		$1^+;1$ state	
	E_x	B(M1)[a]	E_x	B(M1)
	(MeV)	$(10^{-2}\mu_N^2)$	(MeV)	(μ_N^2)
Experiment	12.71	4.0(3)	15.11	2.63(8)
X=1,Y=1	11.80	0.26	13.60	0.89
X=1.5,Y=1	11.03	0.86	13.08	1.89
X=1,Y=0.5	12.30	0.54	13.33	1.12
X=1.5,Y=0.5	11.80	1.65	13.11	2.55

[a]It should be noted that $B(M1;0)$ is enhanced by isospin mixing. One should compare the results of our calculations to the Cohen-Kurath shell model calculation which found that $B(M1;0) = 1.4 \times 10^{-2}\ \mu_N^2$, a value which agrees well with our results, especially for $x = 1.5\ y = 0.5$

TABLE IX. B(E2) strength in ^{12}C

	2$^+$;0 state		2$^+$;1 state	
	E_x	B(E2)	E_x	B(E2)
	(MeV)	e^{2}fm^4	(MeV)	e^{2}fm^4
(e_p, e_n)		(1.5,0.5)		(1.5,0.5)
Expt	4.44	39.9(2.2)	16.11	2.0(0.2)
X=1,Y=1	3.80	80.6	15.78	2.6
X=1.5,Y=1	3.82	75.3	14.44	3.1
X=1,Y=0.5	3.79	80.1	14.86	1.2
			15.58	1.6
X=1.5,Y=0.5	4.11	72.9	14.25	3.8

TABLE X. B(M1) strength in ^{20}Ne.

	1$^+$;1 state			
	E_x	B(M1)[a]	B_σ	B_l
	(MeV)	(μ_N^2)	(μ_N^2)	(μ_N^2)
Expt	11.26	2.02(36)	0.49(6)	0.52(15)
X=1,Y=1	12.40	0.83	0.06	0.43
X=1.5,Y=1	10.73	1.49	0.29	0.47
X=1,Y=0.5	12.19	0.19	0.03	0.24
	12.25	0.73	0.14	0.24
X=1.5,Y=0.5	10.61	1.63	0.35	0.47

[a]It should be noted that only one $M1$ transition is established, but B_σ and B_l can be separated from comparison of (e,e$'$) and (p,p$'$). Constructive interference assumed.

TABLE XI. B(E2) strength in ^{20}Ne

	$2^+;0$ state		$2^+;0$ state	
	E_x (MeV)	$B(E2)$ (e^{2}fm^4)	E_x (MeV)	$B(E2)$ (e^{2}fm^4)
(e_p,e_n)		(1.5,0.5)		(1.5,0.5)
Expt	1.63	340(22)	7.42	<0.83
	7.83	12.0(1.5)	10.27	1.4(4)
X=1,Y=1	2.92	299	9.21	2.9
	11.82	4.4	11.12	4.1
			11.76	0.97
X=1.5,Y=1	2.40	298	7.82	0.95
	10.14	0.7		
	10.75	3.6	9.90	1.3
	11.08	0.4	10.76	3.0
X=1,Y=0.5	2.84	301	9.01	2.6
	11.08	0.4	11.20	3.9
	11.74	3.8	11.65	1.1
X=1.5,Y=0.5	2.37	296	7.61	0.73
	10.05	1.8	9.81	1.7
	10.73	3.0	10.90	2.5

TABLE XII. B(M1) strength in ^{22}Ne

	$1^+;1$ state		$1^+;1$ state		$1^+;1$ state	
	E_x (MeV)	$B(M1)$ (μ_N^2)	E_x (MeV)	$B(M1)$ (μ_N^2)	E_x (MeV)	$B(M1)$ (μ_N^2)
Expt.	5.33	0.43(17)	6.85	1.36(56)	9.18	1.80(7)
(x,y)						
(1,1)	5.70	0.02	6.85	0.14	8.45	1.46
(1.5,1)	7.06	0.46	8.10	0.94	10.61	1.36
(1,0.5)	5.59	0.007	7.76	0.15	8.63	1.80
(1.5,0.5)	6.99	0.79	8.56	1.07	10.95	1.52

TABLE XIII. B(E2) strength in ^{22}Ne.

$(e_p, e_n)^\mathrm{a}$	$2^+;1$ state		$2^+;1$ state	
	E_x (MeV) (1.5,0.5)	$B(E2)$ (e^2fm^4) (1.5,0.5)	E_x (MeV)	$B(E2)$ (e^2fm^4)
Expt	1.27	235(3)	4.46	> 4.6
	6.12	3.5(1.5)	7.64	19(8)
X=1,Y=1	2.27	199	3.10	113
	4.21	2.9	5.85	12.4
X=1.5,Y=1	1.98	289	2.77	7.5
	4.27	8.7	5.03	22.9
X=1,Y=0.5	2.28	243	3.02	67
	4.18	6.5	5.72	13.0
X=1.5,Y=0.5	1.90	290	3.00	0.004
	4.43	7.1	4.88	26.1

[a]Results for the other (e_p, e_n) combinations are significantly worse. It should be noted that there is no experimental evidence for isovector $E2$ transitions.

TABLE XIV. Experimental $B(M1)$ values[a] for ^{24}Mg

E_x MeV	$B(M1)\uparrow$ (μ_N^2)
8.865	0.042 ± 0.011
9.820	0.260 ± 0.040
9.962	1.160 ± 0.150
10.711	3.180 ± 0.300
10.918	0.100 ± 0.020
11.002	0.048 ± 0.011
11.373	0.056 ± 0.013
11 851	0.033 ± 0.008
11.960	0.046 ± 0.010
12.145	0.035 ± 0.012
12.264	0.035 ± 0.009
12.406	0.047 ± 0.010
12.527	0.065 ± 0.011
12.807	0.179 ± 0.018
12.961	0.151 ± 0.012
13.081	0.024 ± 0.011
13.222	0.067 ± 0.009
13.675	0.030 ± 0.008
13.940	0.201 ± 0.014
14.094	0.026 ± 0.008
14.267	0.059 ± 0.009

(a) obtained with the use of the Wildenthal-Brown form factor.

TABLE XV. Experimental Excitation Energies and B_σ values[a] for 1^+ $T = 1$ states ^{24}Mg

E_x MeV	B_σ (μ_N^2)
9.83 ($T = 0$?)	0.29 ± 0.02
9.97	0.38 ± 0.03
10.72	2.75 ± 0.20
12.53	0.60 ± 0.03
12.82	0.85 ± 0.03
12.96	0.37 ± 0.02
13.90	0.56 ± 0.09
16.12	0.31 ± 0.06

(a) Taken from (p, p') data by Crawley $et.al.$ [18].

TABLE XVI. Calculated $B(M1)$ ↑ strengths to $J = 1^+$ $T = 1$ states in ^{24}Mg obtained with the Realistic Interaction ($x = 1.5$ $y = 0.5$).

| E_x | $B(M1)_{physical}$ | B_l | B_σ |
MeV	(μ_N^2)	(μ_N^2)	(μ_N^2)
9.33	2.11	0.19	1.02
9.47	0.64	0.08	0.27
10.50	0.03	0.00	0.03
11.91	0.19	0.07	0.03
12.14	0.23	0.21	0.001
13.49	0.03	0.01	0.008
13.92	0.02	0.24	0.38
14.24	0.02	0.02	0.00
14.59	0.03	0.00	0.05
15.00	0.15	0.12	0.002
Sum(10)[a]	3.45	0.94	1.79
Total Sum[b]	5.43	1.69	4.04

(a) Sum to the lowest 10 states shown in the table.

(b) Sum to all $J = 1^+$ $T = 1$ states in the model space.

TABLE XVII. Calculated $B(M1)$ ↑ strengths to $J = 1^+$ $T = 1$ states in ^{24}Mg obtained with ($x = 1.5$ $y = 0.5$) and $\Delta\epsilon_s = 2$ MeV.

| E_x | $B(M1)_{physical}$ | B_l | B_σ |
MeV	(μ_N^2)	(μ_N^2)	(μ_N^2)
9.40	3.80	0.18	2.32
9.88	0.00	0.05	0.10
11.08	0.08	0.04	0.01
12.08	0.00	0.09	0.14
13.08	0.00	0.18	0.14
13.33	0.06	0.06	0.00
13.98	0.01	0.06	0.13
14.06	0.10	0.07	0.35
14.89	0.42	0.06	0.16
15.19	0.10	0.02	0.03
Sum(10)	4.58	0.81	3.38
Total Sum	6.45	1.63	5.46

TABLE XVIII. B(E2) strength in ^{24}Mg

	$2^+;0$ state		$2^+;0$ state	
	E_x	$B(E2)$	E_x	$B(E2)$
	(MeV)	(e^2fm^4)	(MeV)	(e^2fm^4)
(e_p, e_n)		(1.5,0.5)		(1.5,0.5)
Expt	1.369	435(9)	4.238	33.3(2.4)
	7.349	11.8(4.7)	9.004	3.3(0.6)
X=1,Y=1	2.78	488	4.33	11.9
	9.32	0.6	9.93	3.8
X=1.5,Y=1	2.39	474	3.74	12.8
	7.87	0.8	9.67	3.5
X=1,Y=0.5	2.71	486	4.24	13.4
	9.07	0.8	10.0	2.7
X=1.5,Y=0.5	2.33	479	3.62	4.9
	7.44	1.51	9.31	4.4

TABLE XIX. Experimental $B(M1)$ values[a] for ^{28}Si

E_x	$B(M1) \uparrow$
MeV	(μ_N^2)
10.594	0.19 ± 0.04
10.725	0.11 ± 0.01
10.901	0.90 ± 0.02
11.445	4.42 ± 0.20
12.331	0.87 ± 0.06
14.030	0.37 ± 0.02
15.147	0.23 ± 0.02
15.500	0.26 ± 0.03
17.56	0.18 ± 0.03

(a) Obtained through DWBA analysis.

TABLE XX. Experimental Excitation Energies and B_σ values[a] for 1^+ $T = 1$ states ^{28}Si

E_x	B_σ
MeV	(μ_N^2)
9.72	0.39 ± 0.06
10.59	0.83 ± 0.83
10.73	0.32 ± 0.04
10.82	0.21 ± 0.04
10.90	0.35 ± 0.05
11.16	0.31 ± 0.07
11.45	3.32 ± 0.24
12.33	0.73 ± 0.14
12.99	0.23 ± 0.05
13.35	0.81 ± 0.14
14.03	1.31 ± 0.12
15.15	0.42 ± 0.04
15.50	0.12 ± 0.08
15.80	0.22 ± 0.02
Sum	9.51

(a) Taken from (p, p') data by Crawley $et.al.$ [18].

TABLE XXI. Calculated $B(M1) \uparrow$ strengths to $J = 1^+$ $T = 1$ states in ^{28}Si obtained with the Realistic Interaction $(x = 1.5 \ y = 0.5)$.

E_x	$B(M1)_{physical}$	B_l	B_σ
MeV	(μ_N^2)	(μ_N^2)	(μ_N^2)
9.43	3.90	0.22	2.27
10.54	0.80	0.06	1.31
11.05	0.00	0.03	0.02
12.07	0.02	0.03	0.00
12.65	0.20	0.03	0.41
13.27	0.06	0.14	0.38
13.32	1.39	0.06	0.02
14.11	0.02	0.00	0.02
14.61	0.11	0.06	0.33
14.78	0.05	0.02	0.01
Sum(10)[a]	5.30	0.66	4.77
Total Sum[b]	7.54	1.46	6.97

(a) Sum to the lowest 10 states shown in the table.
(b) Sum to all $J = 1^+$ $T = 1$ states in the model space.

TABLE XXII. Calculated $B(M1) \uparrow$ strengths to $J = 1^+$ $T = 1$ states in ^{28}Si obtained with $(x = 1.5$ $y = 0.5)$ and $\Delta\epsilon_s = 2$ MeV.

E_x	$B(M1)_{physical}$	B_l	B_σ
MeV	(μ_N^2)	(μ_N^2)	(μ_N^2)
10.03	4.61	0.01	4.26
10.79	0.14	0.10	0.00
10.99	0.39	0.00	0.49
11.64	0.76	0.03	1.07
12.71	0.76	0.14	0.25
12.82	0.01	0.00	0.01
13.25	0.15	0.20	0.68
13.64	0.00	0.00	0.01
14.08	0.08	0.04	0.25
14.47	0.17	0.00	0.21
Sum(10)	7.08	0.52	7.24
Total Sum	8.94	1.36	9.04

TABLE XXIII. Experimental $B(E2)$ $(0_1^+ \rightarrow 2_1^+ T)$ in ^{28}Si

E_x	$B(E2)$	T
(MeV)	$(e^2 fm^4)$	
1.779	334(6)	0
7.381	8.5(0.6)	0
7.416	4.5(0.5)	0
7.993	7.7(1.1)	0
8.259	0.73(0.17)	0
9.381	1.3(0.5)	1

TABLE XXIV. Theoretical $B(E2)$ $(0_1^+ \rightarrow 2_1^+ T)$ in ^{28}Si calculated with $x = 1.5$ $y = 0.5$ $(e_p = 1.5$ $e_n = 0.5)$

E_x	$B(E2)$	T
(MeV)	$(e^2 fm^4)$	
2.41	496.7	0
4.64	4.3	0
6.87	1.4	0
8.00	26.7	0
8.59	8.8	1
9.34	0.3	0
9.92	0.7	0

TABLE XXV. Experimental Excitation Energies and B_σ
and $B(M1)$ values[a] for 1^+ $T = 1$ states ^{32}S

E_x	B_σ	$B(M1)$[b]
MeV	(μ_N^2)	(μ_N^2)
7.63	0.42 ± 0.07	
7.92	0.10 ± 0.02	
8.13	1.46 ± 0.19	1.14 ± 0.18
9.66	0.13 ± 0.02	0.69 ± 0.18
11.13	4.08 ± 0.53	2.40 ± 0.22
11.63	2.38 ± 0.35	1.26 ± 0.20
11.88	0.37 ± 0.06	
12.56	0.33 ± 0.06	
13.90	0.24 ± 0.03	
14.88	0.20 ± 0.04	
15.58	0.28 ± 0.05	
15.70	0.16 ± 0.04	
15.84	0.26 ± 0.06	

(a) Taken from (p, p') data by Crawley $et.al.$ [18].
(b) From (e, e') data cited by Crawley $et.al.$ [18].

TABLE XXVI. Experimental Excitation Energies and
$B(M1)$ values[a] for 1^+ $T = 1$ states ^{32}S

E_x	$B(M1)$
MeV	(μ_N^2)
9.66	0.55 ± 0.24
9.98	0.09 ± 0.04
10.45	0.10 ± 0.08
10.90[b]	0.33 ± 0.04
11.16	1.24 ± 0.13
11.50	0.10 ± 0.04
11.65	0.77 ± 0.14
12.19	0.14 ± 0.09
12.65	0.11 ± 0.04
12.98	0.07 ± 0.04
13.41[b]	0.54 ± 0.06
13.78	0.39 ± 0.05
13.97[b]	0.20 ± 0.11
14.45	0.18 ± 0.05

(a) Taken from paper by Petraitis $et.$ $al.$ [20].
(b) May be $M1$ or $M2$.

TABLE XXVII. Calculated $B(M1)$ ↑ strengths to $J = 1^+$ $T = 1$ states in ^{32}S obtained with the Realistic Interaction ($x = 1.5$ $y = 0.5$).

E_x	$B(M1)_{physical}$	B_l	B_σ
MeV	(μ_N^2)	(μ_N^2)	(μ_N^2)
10.05	2.91	0.02	3.37
11.21	5.34	0.02	5.98
13.02	0.68	0.12	0.23
13.85	0.44	0.11	0.98
15.10	0.04	0.06	0.22
15.62	0.13	0.02	0.05
15.93	0.01	0.02	0.01
16.55	0.16	0.02	0.07
17.01	0.27	0.14	0.02
17.31	0.14	0.01	0.23
Sum(10)	10.13	0.53	11.16
Total Sum	10.98	11.52	12.03

TABLE XXVIII. Calculated $B(M1)$ ↑ strengths to $J = 1^+$ $T = 1$ states in ^{32}S obtained with ($x = 1.5$ $y = 0.5$) and $\Delta\epsilon_s = 2$ MeV.

E_x	$B(M1)_{physical}$	B_l	B_σ
MeV	(μ_N^2)	(μ_N^2)	(μ_N^2)
8.58	0.40	0.01	0.53
10.25	4.33	0.00	4.20
10.98	0.11	0.15	0.52
12.08	2.64	0.01	3.02
12.40	0.05	0.01	0.01
13.04	0.18	0.00	0.15
14.22	0.02	0.06	0.15
14.83	0.12	0.01	0.22
15.10	0.02	0.11	0.04
15.17	0.95	0.04	0.59
Sum(10)	8.81	0.41	9.45
Total Sum	10.19	1.24	10.91

TABLE XXIX. B(E2) strength in ^{32}S

	$2^+;0$ state		$2^+;0$ state	
	E_x	$B(E2)$	E_x	$B(E2)$
	(MeV)	(e^{2}fm^4)	(MeV)	(e^{2}fm^4)
(e_p,e_n)		(1.5,0.5)		(1.5,0.5)
Expt	2.23	306(9)	4.49	58.1(6.9)
	5.55	3.9(1.0)	7.12	¿ 1.0
X=1,Y=1	3.00	505	4.63	10.1
	9.9	0.8	11.3	1.6
X=1.5,Y=1	4.45	28.3	5.39	41.1
	9.4	0.6	10.4	12.2
X=1,Y=0.5	3.19	460	4.62	16.2
	9.5	0.7	10.7	2.8
X=1.5,Y=0.5	5.77	186	6.32	58.1
	12.6	3.9	11.3	27.3

TABLE XXX. B(M1) strength in ^{36}Ar. Only summed strength is discussed because of the large error bars of the (e,e$'$) experiment.

	B(M1) (μ_N^2)
Expt	2.65 ± 0.1
X=1,Y=1	2.30
X=1.5,Y=1	4.09
X=1,Y=0.5	2.68
X=1.5,Y=0.5	4.63

TABLE XXXI. B(E2) strength in ^{36}Ar

	2$^+$;0 state		2$^+$;0 state	
	E_x (MeV)	B(E2) (e^{2}fm^4)	E_x (MeV)	B(E2) (e^{2}fm^4)
(e_p, e_n)		(1.5,0.5)		(1.5,0.5)
Expt	1.97	340(40)	4.44	14.0(2.5)
X=1,Y=1	2.85	312	7.10	2.7
X=1.5,Y=1	2.23	247	5.96	6.3
X=1,Y=0.5	2.77	301	6.96	5.0
X=1.5,Y=0.5	2.19	236	5.88	3.6

TABLE XXXII. B(E2) strength in ^{36}Ar continued

	2$^+$;0 state		2$^+$;1 state	
	E_x (MeV)	B(E2) (e^{2}fm^4)	E_x (MeV)	B(E2) (e^{2}fm^4)
(e_p, e_n)		(1.5,0.5)		(1.5,0.5)
Expt	4.95	< 21.4	6.61	2.4(0.9)
X=1,Y=1	9.39	2.2	8.56	3.6
X=1.5,Y=1	8.19	20.8	6.34	8.0
X=1,Y=0.5	8.81	5.4	8.19	4.6
X=1.5,Y=0.5	8.51	30.0	6.28	8.5

TABLE XXXIII. $B(E2)$ strength in ^{44}Ti

	2$^+$;0 state		2$^+$;0 state	
	E_x (MeV)	B(E2) (e^{2}fm^4)	E_x (MeV)	B(E2) (e^{2}fm^4)
Expt	1.08	540(140)	2.53	14.2(2.4)
X=1,Y=1[a]	2.08	924	6.12	5.0
X=1.5,Y=1	1.67	913	5.65	2.1
X=1,Y=0.5	2.02	929	6.06	4.0
X=1.5,Y=0.5	1.65	908	5.68	2.3

(a) The calculations were done with $e_p = 1.5$, $e_n = 0.5$.